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SHORT ARTICLE

# Description of Physical Status and Definition of Wave Function and Derivation of Klein-Gordon Lagrangian Using New Mathematical 4 Rules and Deviation from the Shortest Path of Fermion

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## Abstract:

In this paper 4 new rules of mathematics will be applied to describe the movement status of such a particle like boson and fermion and derive Klein-Gordon Equation. In the first section, the 4 rules themselves are defined and explained. In the next section, the 4 rules are applied to Einstein's special relativity by an example. In the next section, the 4 rules are used to define fermion's wave function using the shortest path and deviation from them. Finally, using definition of Energy and momentum, Klein-Gordon Lagrangian which can be used to derive Klein-Gordon Equation is derived using above 4 rules.

Keywords: Klein-Gordon Lagrangian, Four new rules of Mathematics, Wave function, Physical Status

## Introduction

Assume that a certain physical quantity is assigned to objects. then The 4 rules which will be applied to the quantity are described as follows :

1. If the 'change' or 'difference' of physical quantity while the change of spacetime is infinitesimal is the most symmetrical over all the other changes, then real infinitesimal number $\epsilon$ will be assigned to this change. If the change is in the time forward direction, then the number $\epsilon$ must be positive number, and if it is in the time backward direction, negative real number must be assigned to the change.

2. If the 'change' or 'difference' of physical quantity while the change of spacetime is infinitesimal is the change which is not allowed by Einstein's Relativity (the most non-symmetrical), then the pure imaginary number $i\epsilon$ will be assigned to this change. This change is spacelike movement of certain particle, or the deviation of the object's path from the General Relativity's shortest path, for example.

Assume there are 2 status of an object and each is described by 2 numbers of physical quantities each. Then 2 following rules must be applied to these 2 numbers.

3. The Rule of Addition: If the physical status (represented by complex number C) consists of 2 distinguishable status which is represented by the complex number A, B, then C is obtained by Addition of A and B. The word distinguishable status means that there is no uncertainty between A and B when they forms physical status C.
4. The Rule of Multiplication: If the physical status (represented by complex number C) consists of 2 indistinguishable status which is represented by the complex number A, B, then C is obtained by multiplication of A and B. The word indistinguishable status means that there is non-zero uncertainty between A and B when they forms physical status C.

If there are 1 object A in one closed universe and the physical quantity to be discussed is 4 spacetime point of it, for example. A's 4 spacetime is

$$(t_A, \vec{x}_A),$$

the change to be mentioned is the change or difference of a spacetime or 'infinitesimal path' or 'path' to be short.

First apply rule 1 and 2 to this particle's path. The most symmetrical infinitesimal change over all the other 'infinitesimal change' of spacetime point is timelike movement (no space change). In this case, A movement can be described by following 2 4 spacetime coordinates (real number).

$$\mathrm{A} : (dt_A, d\tau_A)$$

where t is change of time of observer observing A objects, and $\tau$ is change of time of A

In contrast, relativistically non-allowed movement (non-symmetrical) can be spacelike movement (traveling in light speed) (If A has non zero rest mass). Therefore, this movement must be described by pure imaginary number by following way :

$$\mathrm{A} : (dt_A, id\vec{x}_A)$$

Assuming A is observed by the same observer, and A's movement is between the most symmetrical and non-symmetrical, numbers of movement mentioned above must be added by following way because each spacelike and timelike movement can be easily distinguishable (there is no uncertainty between spacelike and timelike movement).

$$\mathrm{A} : (dt, id\vec{x}_A + d\tau_A)$$

every movement of A can be described by this number.

In contrast, the movement by A of dt and dτ of the same directions and the opposite directions cannot be distinguishable (There is uncertainty between dt and dτ of the same sign and the different sign), therefore the following 2 numbers must be multiplied ( each number consists of 4 components(vector), so dot product must be applied.) dt and dτ the same direction:

$$(dt, id\vec{x}_A + d\tau_A)$$

dt and dτ the opposite direction :

$$(dt, id\vec{x}_A - d\tau_A)$$

If above 2 4 spacetime movements are dot producted,

$$dt^2 + (id\vec{x}_A + d\tau_A) \cdot (id\vec{x}_A - d\tau_A) = dt^2 - dx_A^2 - d\tau_A^2.$$

If movement of A is consistent,(inertial reference frame) then

$$dt^2 - dx_A^2 - d\tau_A^2 = 0$$

$$d\tau_A^2 = dt^2 - dx_A^2$$

.

Now apply this method to the change of infinitesimal path of some objects.

Assume there are 2 objects in Fig 1. According to general relativity by Einstein, every object is assigned to travel on the shortest path in spacetime. Each of 2 object a, b is assigned AA' and BB' as the shortest path of them, respectively. On the way of each one's shortest path, each one meets each other at the same 4 spacetime point named C. Meeting point is shielded by "box", so that no one can see the things inside it.

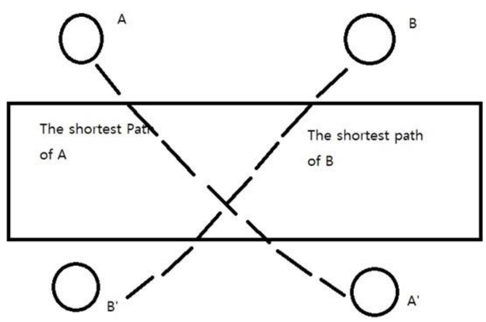


Figure 1. The picture describing two objects each on the shortest path of their own, meeting each other at the point C shielded by black box

There are 3 things that can happen to the 2 objects.

Case 1: a, b ignore each other at C

Case 2: a, b swap their path.

Case 3: a, b each travels to their own new paths.

There is actually a kind of particle in the universe which is in the indistinguishable status between case 1 or 2 above in this situation, especially if the 2 particles (a, b in this case) are exactly identical to each other. That's because no matter whether they ignore (case 1), or they swap(case 2), no body knows what happens at C and the final result is always the same. This phenomenon will be described by the word "physically continuous". This kind of particle is called fermion. There are other kinds of (mixed of case 1 and 3) particles called boson, but the focus will be on fermion in this paper.

If object a travels on AA' and b on BB' which are the most symmetrical path (case 1), then the positive real number must be assigned, However, if a and b change their path after meeting each other at point C (case 2), then pure imaginary number must be assigned to their path 'status'. Their actual movement in the picture above is that they start with the shortest path and at the point C, they are mixed together to form indistinguishable status between object a and b. Their final status after certain period of time is that the object a and b hardly distinguishable, but the status of them, whether they belong to case 1, case 2 is distinguishable. Therefore, their final status is

$$\epsilon + i\epsilon$$

in complex number.($\epsilon$ means the case 1, and $i\epsilon$ means the case 2)

In this way fermion always deviates from their own spacetime shortest path every time, and their path 'status', which stands for this deviation will be called 'wave function' in this letter.

Every object with wave function $\psi$ is combined with the wave function $\bar{\psi}$

which forms indistinguishable status with each other and travels on the shortest path. Their final status after certain period of time must be represented by multiplication of $\psi$ and $\bar{\psi}$

In this case, $\bar{\psi}$ is called conjugate wave function of wave function $\psi$. And

$$\bar{\psi}_d \psi_d$$

value in the equation

$$\bar{\psi}_d \psi_d \times V = \bar{\psi}\psi$$

(V space volume) is called wave(probability) density of the wave function. This movement can be simplified by the wave function that starts with $\psi$ and ends up with $\bar{\psi}$ on their path or movement.

Another case considered is the collision which can be discussed with object a which starts with complex number wave function $\psi_a$ and ends up with $\bar{\psi}_a$, and object b which starts with complex number wave function $\bar{\psi}_b$ and ends up with $\psi_b$ and on their ways they meet each other at point C like the picture described above. As pointed out in the paragraph above, at the point C, they are mixed together to form indistinguishable status and finally after certain period of time they are only distinguished by the status of whether they are swapped or not. The status of non-swapped can be described by the wave function

$$\bar{\psi}_a \psi_b,$$

and swapped wave function is

$$\bar{\psi}_b \psi_a.$$

As mentioned earlier, assign positive real number to non-swapped one and pure imaginary number to swapped one. Therefore total wave function is

$$\bar{\psi}_a\psi_b + i\bar{\psi}_b\psi_a$$

.

For symmetrical reason, object a which starts with complex number wave function $\bar{\psi}_a$ and ends up with $\psi_a$, object b which starts with $\psi_b$ ends up with $\bar{\psi}_b$. Then after meeting at point C, the total wave function is

$$\psi_a\bar{\psi}_b - i\psi_b\bar{\psi}_a$$

(Negative sign is because the swapping is done in opposite direction to the former one)

If one subtract two wave function with each other, the result becomes:

$$\bar{\psi}_a\psi_b - \psi_a\bar{\psi}_b + i(\bar{\psi}_b\psi_a + \psi_b\bar{\psi}_a)$$

The latter part (imaginary part) of this result represents the deviation from the shortest path by objects and will be called commutator in this paper.

The subtraction can be done in opposite direction, therefore there are 2 versions of commutator and each one represents 2 spin of electron wave function.

One can consider again another kind of physical quantity, which can be represented by P in the equation

$$k\bar{\psi}\delta\psi = P\bar{\psi}\psi$$

, and will be called "Energy Status." One can also assign pure imaginary number to the most non symmetrical(violating relativity) quantity of P, and assign real number to the most symmetrical quantity. Here the most symmetrical quantity is the one in which $\delta\psi$ and corresponding conjugate $\delta\bar{\psi}$ satisfy following equation,

$$\bar{\psi}\psi = (\bar{\psi} + \delta\bar{\psi})(\psi + \delta\psi).$$

and assign imaginary number to the one which doesn't satisfy the above equation. For the quantity of P to have real number in the most symmetrical situation, the constant k must have the value -ih where h is real number, because

$$\bar{\psi}\delta\psi$$

itself is pure imaginary number. Especially if

$$-ih\bar{\psi}\,\partial\psi = \partial tE\bar{\psi}\psi,$$

And $\partial\psi$, $\partial t$ means the variation without any change of space positions and E is positive real number (h is Planck Constant), then E is called Energy. Similarly if

$$-ih\bar{\psi}\,\partial\psi = \partial x_n P_n\bar{\psi}\psi$$

, and $P_n$ is real number, then $P_n$ is momentum in space $x_n$ direction. In these cases, Energy and momentum is conserved. In contrast, if E and $P_n$ also have imaginary component, then energy and momentum is not conserved.(Energy and momentum conservation law is violated)

Now assume there are N identical infinitesimal fermions each of which has the value of energy status $P_i$,, energy $E_i$and momentum $P_{ni}$ as a complex number. There are 2 ways to represent this system's energy status. The one of those is:

$$\Sigma(P_i\bar{\psi}\psi)$$

, which represents the summarization(integration) of each infinitesimal wave function's energy status. The other one is:

$$\mathrm{P}'\bar{\psi}'\psi'$$

, where $\psi'$ and $\bar{\psi}'$ are the wave functions which correspond to the center of entire system.

$RE(P_i\bar{\psi}\psi)$ (real part of $P_i\bar{\psi}\psi$) in each term of $\Sigma(P_i\bar{\psi}\psi)$ is not zero as long as each infinitesimal fermion in the system is physically continuous to each other. It represents actual energy (or momentum) of the fermion systems. Therefore if The 1st physics law is defined as following - Every infinitesimal existence which has energy is always physically continuous to each other (The law of wave)- Then

$$RE\left(\Sigma(P_i\bar{\psi}\psi)\right) \neq 0$$

is the condition of satisfaction of this 1st law.

$$IM\left(\Sigma(P_i\bar{\psi}\psi)\right)$$

(imaginary part of $\Sigma(P_i\bar{\psi}\psi)$) and $RE(\mathrm{P}'\bar{\psi}'\psi')$ are the things which is related to the wave density(probability) of the entire system.

$$IM(\Sigma(P_i\bar{\psi}\psi)) + \mathrm{P}'\bar{\psi}'\psi' = \Sigma\bar{\psi}\psi.$$

(If N is infinite, then Σ must be replaced by integration). If

$$\Sigma\bar{\psi}\psi = 1,$$

it means the satisfaction of the following physics law: Every existence's integration(summarization) of wave density(probabilistic density) over every part of physically continuous must be integer. Here the integer means the number of electron according to the current Quantum Mechanics.

$$IM(P_i{}'\bar{\psi}'\psi')$$

is the part which is related to perturbation of energy and momentum.

There are only 2 kinds of such systems which consists of N identical infinitesimal fermions: macroscopic and microscopic system. Macroscopic system is the one which violates the law of wave and satisfies the law of particle.

$$((RE\left(\Sigma(P_i\bar{\psi}\psi)\right) = 0 \text{ and } \Sigma\bar{\psi}\psi = n \text{ (n is any integer)}).$$

Microscopic system is the one which satisfies the law of wave and violates the law of particle.

$$((RE\left(\Sigma(P_i\bar{\psi}\psi)\right) \neq 0 \text{ and } \Sigma\bar{\psi}\psi \neq n \text{ (n is any integer))}$$

Microscopic system always violates energy and momentum conservation law : The perturbation represented by

$$IM(P_i{}'\bar{\psi}'\psi')$$

is always non-zero for every microscopic system. Macroscopic system must satisfy above 2 physics laws (energy and momentum conservation law), but in reality it is false, because there is always a possibility as at least the part of one system can be physically continuous to the other system's part. Therefore, if

$$RE(P_i{}'\bar{\psi}'\psi') < 1/2$$

, the value of

$$IM(P_i{}'\bar{\psi}'\psi')$$

is quite small, and this situation is considered the satisfaction of momentum and energy conservation law. Instead, we adopt the interaction with boson field for the compensation of this perturbation(non-conservation), which is photon represented by A.

Actual energy

$$\Sigma(P_i\bar{\psi}\psi)$$

of the whole fermion system can be obtained by letting $\Sigma\bar{\psi}\psi => 0$, and which is $E_f$. This energy satisfies the following equation:

$$-ih\bar{\psi}\,\partial\psi = \partial t E_f\bar{\psi}\psi.$$

Energy perturbation $E_p$ is obtained by the following equation:

$$IM(P_i{}'\bar{\psi}'\psi') = \bar{\psi}'A\psi' = E_p\bar{\psi}'\psi'.$$

Total energy is

$$E_f\bar{\psi}\psi + E_p\bar{\psi}\psi = -ih\bar{\psi}\left(\frac{\partial\psi}{\partial t}\right) + \bar{\psi}A\psi.$$

By the same calculation, Total momentum is

$$P_{nf}\bar{\psi}\psi + P_{np}\bar{\psi}\psi = -ih\bar{\psi}(\partial\psi/\,\partial x_n) + \bar{\psi}A\psi$$

In the fermion case, the total macroscopic system can be described by following vectors:

$$(-ih\bar{\psi}(\partial\psi/\,\partial t) + \bar{\psi}A\psi, -ih\bar{\psi}(\partial\psi/\,\partial x_n) + \bar{\psi}A\psi)$$

$$(-ih\psi(\partial\bar{\psi}/\,\partial t) - \bar{\psi}A\psi, +ih\psi(\partial\bar{\psi}/\,\partial x_n) + \bar{\psi}A\psi)$$

$$(-ih\bar{\psi}(\partial\psi/\,\partial t) + \bar{\psi}A\psi, +ih\bar{\psi}(\partial\psi/\,\partial x_n) + \bar{\psi}A\psi)$$

$$(-ih\bar{\psi}(\partial\psi/\,\partial t) - \bar{\psi}A\psi, -ih\bar{\psi}(\partial\psi/\,\partial x_n) + \bar{\psi}A\psi)(n = 1,2,3)$$

The 1st 2 vectors represent fermion state of one spin, and the next 2 vectors represent fermion state of the other spin. The 1st 2 vectors compose indistinguishable one orthogonal spin state, therefore, one can multiply the 2 vectors with each other.

$$-h^2(\partial\psi/\partial t)^2 - A^2 + \Sigma h^2(\partial\psi/\partial x_n) + A^2 = m\bar{\psi}\psi$$

if the 2

$$A^2$$

have the same values, then

$$-(\partial\psi/\partial t)^2 + (\partial\psi/\partial x)^2 = m\bar{\psi}\psi$$

after going through some of manipulation, the above equation can lead to the non-variation of Klein-Gordon Lagrangian

$$\mathrm{L} = \frac{1}{2}\,\partial_\mu\psi\,\partial^\mu\psi - \frac{1}{2}m\bar{\psi}\psi$$

(the Lagrangian that leads to Klein-Gordon Equation)

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