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RESEARCH ARTICLE

Definition of Mass and Production of Equations of General Relativity

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Abstract:

From the measurements of the anisotropies of the cosmic background radiation at the present time, we get a value for the density parameter (Ω(t)) near to unit, i.e. $\Omega(t) \sim 1$.
The value of the density parameter determines if the Universe is open (Ω(t) $<$ 1), flat (Ω(t) $=$ 1) or closed (Ω(t) $>$ 1)). This result forces us to assume that the boundary of the Universe is a 2D flat space, i.e. the R^2, since its interior is a 3D space as we conceive it.
The R^2 space is characterized by isotropy and homogeneity. It is a simply connected space and that it does not exhibit any particular characteristic anywhere. These attributes are expressed by a circle of an infinite radius in the center of which is an observer, at every point in the Universe.
Since circle is the geometric object from which all other conic sections produced, then we shall examine the equations that characterize them and the consequences of their mappings in the interior of the Universe through one to one correspondence.

Keywords: Conic sections, Affine connections, Differential manifold

1. Definition of a differential manifold

In this paper, the general relations that determine conic sections in a *curved differential manifold* M is applied by observers of the 3D space (free observers). From this process, they will find equations of the same form as Einstein's equations of general relativity, *without using the principle of equivalence*. So, those equations, free from the equivalence principle, are used in all regions of physics and not only to gravity.

In the founding of Classical Mechanics (Krikos, 2019) it was referred that a point circle of R^2 is depicted as a material point in 3D space generating a local anisotropy in its neighborhood, due to the mass attributed to it. In other words, the neighborhood of this material point exhibits curvature. In order to calculate the various physical quantities, a coordinate system that is only locally approached by a Cartesian coordinate system should be defined.

If a second material point located in the neighborhood of the first, with a different mass from that of the first, correlates with the first one. In the case of Classical Mechanics it was considered that 3D space does not change its basic characteristics, that is, that it is still isotropic and homogeneous. The second material point creates its own neighborhood in which also a curvature, other than that of the neighborhood of the first material point, is displayed. Thus a different coordinate system other than the coordinate system of first material point, is required, in order to calculate the different physical quantities in this neighborhood. Since these two material points are correlated, then this correlation should be expressed through the correlations of both coordinate systems.

A collection of such material points defines a *differential manifold* M whose basic characteristic is that it is covered by a set of coordinate neighborhoods that each neighborhood has the same number of coordinates as each other. The basic property of two different coordinate systems is that in a common region they are related to a differentiable transformation of a class not less than 1. (Willmore, 2012).

2. Equations of conic sections in a differential manifold

The following relations

$$\mathbf{L} = \nabla \times \mathbf{v} \tag{2.1}$$

$$\mathbf{R} = \nabla \times \mathbf{L} - \nabla\left(\frac{k}{r}\right) \tag{2.2}$$

are the relations through which vector fields L and R are defined on a 3D space, leading to Maxwell's equations in vacuum (Krikos, 2018). These relations will be generalized, writing them as relations between differential forms. The reason we make this generalization is to find their expressions on a flat 3D space, and then to have them transferred to a curved 3D differential manifold. From the general definition of the grad and the curl (William Schulz and Alexia Schulz, 2012), we get

$$curl = \Phi^{-1} \circ \star \circ d \circ \Phi$$

$$grad = \Phi^{-1} \circ d$$

The symbol o denotes the composition of quantities d, Φ, $\star$. The symbol $\star$ is an operator (*Hodge star operator*). The quantities d, Φ, $\star$ and their compositions, that are expressed through *curl and grad*, are independent of the choice of coordinate systems. We write relations (2.1), (2.2) as follows:

$$\mathbf{L} = curl(\mathbf{v})$$

$$\mathbf{R} = curl(\mathbf{L}) + grad(\tilde{\Phi})$$

According to the definitions of curl, grad, we get

$$\mathbf{L} = \Phi^{-1} \star d\Phi(\mathbf{v})$$

$$\mathbf{R} = \Phi^{-1} \star d\Phi(\mathbf{L}) + \Phi^{-1} d(\tilde{\Phi})$$

where $\tilde{\Phi} = -\frac{k}{r}$ belongs to $\Lambda^0(\mathbb{R}^3)$, and $\Lambda^0(\mathbb{R}^3)$ is the set of all functions of $\mathbb{R}^3$. We act on the two members of the above relations from the left, with the function Φ, so we get

$$\Phi(\mathbf{L}) = \star d\Phi(\mathbf{v})$$

$$\Phi(\mathbf{R}) = \star d\Phi(\mathbf{L}) + d(\tilde{\Phi})$$

We act from the left, with the operator $\star$ on the above relations, so we get

$$\star\Phi(\mathbf{L}) = \star\star\, d\Phi(\mathbf{v})$$

$$\star\Phi(\mathbf{R}) = \star\star\, d\Phi(\mathbf{L}) + \star d(\tilde{\Phi})$$

From the general relation

$$\star\star\,\omega = (-1)^{r(n-r)}\omega, \text{ with } \omega \text{ belongs to } \Lambda^r(V^*)$$

where $\omega = d\Phi(\mathbf{v})$, n is the dimensions of V* and the $d\Phi(\mathbf{v})$ belongs to the set $\Lambda^2(\mathbb{R}^3)$ we get

$$\star\Phi(\mathbf{L}) = d\Phi(\mathbf{v})$$

$$\star\Phi(\mathbf{R}) = d\Phi(\mathbf{L}) + \star d(\tilde{\Phi})$$

We define the forms

$$L = \star\Phi(\mathbf{L}), R = \star\Phi(\mathbf{R}), \tilde{D} = \star d(\tilde{\Phi})$$

$$u = \Phi(\mathbf{v}), V = \Phi(\mathbf{L})$$

From the above definitions, relations (2.1), (2.2) are written through differential forms, as

$$L = du \tag{2.3}$$

$$R = dV + \tilde{D} \tag{2.4}$$

The $L, R, \tilde{D}$ are 2-forms, i.e. belonging to the set $\Lambda^2(\mathbb{R}^3)$, and u, V are 1-forms, i.e. belonging to the set $\Lambda^1(\mathbb{R}^3)$. In $\mathbb{R}^3$, we can use the natural coordinate system (x^i) or any other curvilinear coordinate system (v^i), where the v^i are not necessarily rectangular.

3. Definition of mass

Since the neighborhood of a point of a differential manifold M resembles with a neighborhood of the Euclidean space, then a depiction between them can be defined. In the neighborhood of this point of M is also depicted the relation (2.4), that is

$$R = dV + \tilde{D}$$

In the case of applying this relation to a circle in $\mathbb{R}^2$, the quantity dV expresses the correlation of the center of the circle with a point on its periphery.

This correlation results from the change of the tangent on two infinitesimally close points of the circumference of the circle, defining the "acceleration" of each point. The quantity $\tilde{D}$ gives the measure of this correlation for the free observers, because it is the depiction of the term $\mathbf{r}_C(\frac{k}{r_c})$ which contains the k from which the mass or charge arises. Based on these observations, free observers in a curved differential manifold, follow a corresponding procedure as follows:

On each point of a curved differential manifold, a matrix V is attached, whose elements are expressed by the basis (dx^i) of the 1-forms, resulting from the coordinate system (x^i), that is associated to this point. The matrix elements of V, are expressed as (Willmore, 2012)

$$V^i_j = L^i_{jk} dx^k \tag{3.1}$$

The matrix V of a point of a curved differential manifold is called *affine connection*. The coefficients L^i_{jk} belong to the set Λ^0(M), where the set Λ^0(M) includes all functions of M ,are called coefficients of the affine connection. On each point of a curve of a neighborhood, is attached a matrix V , whose elements are homogeneous of first degree in dx^k, which is resulted from an isomorphic mapping between the tangent spaces of two points belonging to this curve. The matrix V is the mapping of an "affine connection" of a point of $\mathbb{R}^2$, to an affine connection that is attached to a point of a differential manifold M. The elements dV^i_j of dV in equation (2.4) are 2-forms, because they are expressed, as

$$dV^i_j = dL^i_{jk} \wedge dx^k = (\partial_p L^i_{jk}) dx^p \wedge dx^k \quad (3.2)$$

The quantity dV corresponds to the variation of the tangent, that is, the acceleration that correlates the center of a circle and a point on its circumference and characterizes that point. We will now examine how $\tilde{D}_\cdot$, in the right-hand member of equation (2.4), is expressed in each neighborhood of a differential manifold.

Free observers call this neighborhood of a differential manifold *central neighborhood* in correspondence with the circle. At each point of the central neighborhood, the affine connection V and its variation dV have already been attached. We consider all the possible curves passing through every point of this neighborhood that connect it to all points of this neighborhood. Since the points of each neighborhood of the manifold M are characterized by a matrix V and correlate with each other, then the affine connections of the points of the central neighborhood are correlated with each other.

In order those correlations to be expressed mathematically, we construct a space with base elements $dx^r \wedge dx^q$, in which are defined 2-forms. This base of 2-forms, results from the composition of the dual spaces with bases 1-forms, of the tangent spaces of neighboring points of a central neighborhood. Accordingly, the matrix elements of $\tilde{D}_\cdot$, express all possible interactions of any two points of a central neighborhood. So, the matrix elements of $\tilde{D}_\cdot$ can be written as a sum of all affine connections V^i_s of all points of this neighborhood, as follows:

$$\tilde{D}^i_j = V^i_s \wedge V^s_j = L^i_{sr} dx^r \wedge L^s_{jq} dx^q \quad (3.3)$$

where the summation is done with respect to s. Thus, to the matrix elements dV^i_j, are added the matrix elements of $\tilde{D}_\cdot$. Through equation (3.3), is defined the microscopic concept of a mass-source, which is expressed by $\tilde{D}^i_j$, in a central neighborhood. This procedure is similar to that followed in the derivation of the equations of Maxwell, where there $\tilde{D}_\cdot$ as a charge-source (Krikos, 2019).

4. Equations of General Relativity or general equations

Having determine the matrix elements dV_j^i , $\tilde{D}_j^i$ of the terms dV, $\tilde{D}$ of the right member of equation (2.4), the question that is raised is what express the matrix elements Ri of R, in the left member of this relation. We write the relation (2.4) through the components of a mixed tensor as follows

$$R_j^i = dV_j^i + V_s^i \wedge V_j^s \tag{4.1}$$

Substituting the dV_j^i and $V_s^i \wedge V_j^s$ from Equation(3.2) and (3.3), to equation (4.1), we get

$$R_j^i = dL_{jk}^i \wedge dx^k + L_{sr}^i dx^r \wedge L_{jq}^s dx^q$$

Due to the *interchange rule*, i.e.

$$dx^r \wedge dx^q = -dx^q \wedge dx^r$$

we take

$$R_j^i = (\partial_p L_{jk}^i - \partial_k L_{jp}^i) dx^p \wedge dx^k + (L_{sr}^i L_{jq}^s - L_{rs}^i L_{qj}^s) dx^r \wedge dx^q$$

Changing the dummy indices of the above equation properly, we get the relation

$$R_j^i = (\partial_p L_{jk}^i - \partial_k L_{jp}^i + L_{sp}^i L_{jk}^s - L_{ps}^i L_{kj}^s) dx^p \wedge dx^k \tag{4.2}$$

The expression in parenthesis denotes the components of the *curvature tensor* of the affine connection of a point, i.e.

$$L_{jkp}^i = \partial_p L_{jk}^i - \partial_k L_{jp}^i + L_{sp}^i L_{jk}^s - L_{ps}^i L_{kj}^s \tag{4.3}$$

We can use a symmetric affine connection, for a point of a central neighborhood (*central point*), where the coefficients are the *Christoffel symbols*, i.e. the symbols Γ_{jk}^i , which are defined as

$$\Gamma_{jk}^i = \frac{1}{2}(L_{jk}^i + L_{kj}^i) \tag{4.4}$$

By this definition we get $\Gamma_{jk}^i = \Gamma_{kj}^i$. The torsion tensor is defined as

$$T^i_{jk} = \frac{1}{2}(L^i_{jk} - L^i_{kj}) \tag{4.5}$$

For a symmetric affine connection the torsion tensor is zero because

$$T^i_{jk} = \frac{1}{2}(\Gamma^i_{jk} - \Gamma^i_{kj}) \text{ and } \Gamma^i_{jk} = \Gamma^i_{kj}$$

Adding the members of the relations (4.4) and (4.5), we get

$$L^i_{jk} = \Gamma^i_{jk} + T^i_{jk}$$

Substituting the L^i_{jk} in Eq.(4.3), we take

$$L^i_{jkl} = (R_C)^i_{jkl} + (R_T)^i_{jkl}$$

where

$$(R_C)^i_{jkl} = \partial_k \Gamma^i_{jl} - \partial_l \Gamma^i_{jk} + \Gamma^n_{jl}\Gamma^i_{nk} - \Gamma^n_{jk}\Gamma^i_{nl} \tag{4.6}$$

$$(R_T)^i_{jkp} = T^i_{jl,k} - T^i_{jk,l} + T^i_{nl}T^n_{jk} - T^i_{nk}T^n_{jl} - 2T^i_{jn}T^n_{kl} \tag{4.7}$$

with $T^i_{jl,k} = \partial_k T^i_{jl}$. Due to the symmetric affine connection, the torsion tensor is zero, so we only get

$$(R_T)^i_{jkp} = 0$$

Acting through the operator d on both members of equation (2.4), and using the lemma of Poincaré ($ddV = 0$), we take the relation for the matrix elements of the R, $\tilde{D}$, i.e.

$$dR^i_j = d\tilde{D}^i_j \tag{4.8}$$

In order to have the same physical units the two members of equation (4.8), we write

$$\tilde{D} = kT$$

where k is a constant with appropriate units. If $\tilde{D}$ is a conserved quantity, then $d\tilde{D}^i_j = 0$ so, from equation (4.8) we get

$$dR^i_j = 0$$

The above relation leads us to Bianchi identities. We write equation (4.2) as

$$R^i_j = R^i_{jkp} dx^p \wedge dx^k \tag{4.9}$$

where R^i_j are the components of the curvature tensor, with respect to the coefficients Γ^i_{jk} of a symmetric affine connection. Acting by the operator d on equation (4.9), then we get

$$dR^i_j = R^i_{jkp,q} dx^q \wedge dx^p \wedge dx^k = 0 \tag{4.10}$$

where $R^i_{jkp,q} = \frac{\partial R^i_{jkp}}{\partial q}$. Doing a cyclic rotation of the indices *k, p, q* then, because of even number of permutations of each differential with the other two, we take the relationship

$$dR^i_j = (R^i_{jkl,m} + R^i_{jlm,k} + R^i_{jmk,l}) dx^m \wedge dx^k \wedge dx^l = 0$$

From these relations, are taken the identities of Bianchi, i.e.

$$R^i_{jkl,m} + R^i_{jlm,k} + R^i_{jmk,l} = 0 \tag{4.11}$$

Initially, is contracted the left member of equation (4.11), with respect to the indices *i, l* and then from this expression, is lifted the index *j* so, we get (Pathria, 2003)

$$-R^j_{k,m} + R^j_{m,k} + R^{jl}_{mk,l} = 0$$

In this relation, the indices *j, k* are contracted so, we get

$$(R^j_m - \frac{R}{2}\delta^j_m)_{,j} = 0$$

where R is the scalar curvature. From this relation we define the *Einstein's tensor* as

$$G^i_j = R^i_j - \frac{1}{2}\delta^i_j R \tag{4.12}$$

or for covariant components, is taken the relation

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu}(\frac{R}{2}) = 0 \tag{4.13}$$

So, if $\tilde{D}$ is a conserved quantity, then are derived Einstein's equations for gravity

$$G_{\mu\nu} = \tilde{D}_{\mu\nu}$$

or

$$R_{\mu\nu} - g_{\mu\nu}(\frac{R}{2}) = kT_{\mu\nu} \quad (4.14)$$

From the equation (4.8), we will find the *equations of Einstein or general equations*. We write equation (4.8) as

$$d(R^i_j - kT^i_j) = 0$$

The quantity in parenthesis should be a constant, i.e.

$$R^i_j - kT^i_j = \Lambda^i_j \quad (4.15)$$

For the covariant components of the tensors of the equation (4.15) we obtain the equations

$$R_{\mu\nu} = kT_{\mu\nu} + g_{\mu\nu}\Lambda \quad (4.16)$$

The constant Λ is called *Λ-cosmological constant*. Equations (4.14) or (4.16) are raised *through the reformulation of the definition of mass without using the principle of equivalence*. The equations (4.16), are called general, because they can be applied in any physical scale, and not only to gravity. In gravity's scale, the equations (4.14), which are named *Einstein's equations of General Relativity*, are applied. The constant k depends on the particular problem we are facing. We can apply these equations in the case of strong interactions, finding the equations that govern the motion of quarks inside a nucleon.

$$R_{\mu\nu} = kT_{\mu\nu} + g_{\mu\nu}\tilde{\Lambda} \quad (4.17)$$

Add and subtract to the right-hand side of this equation the quantity $g_{\mu\nu}(\frac{R}{2})$ where R is the scalar curvature, so we get

$$R_{\mu\nu} = kT_{\mu\nu} + g_{\mu\nu}(\frac{R}{2}) + g_{\mu\nu}(\tilde{\Lambda} - \frac{R}{2}) \quad (4.18)$$

or

$$R_{\mu\nu} = kS_{\mu\nu} + g_{\mu\nu}\Lambda \tag{4.19}$$

where the quantity $kT_{\mu\nu} + g_{\mu\nu}(\frac{R}{2})$ becomes

$$k[T_{\mu\nu} - \frac{R}{2}g_{\mu\nu}]T^{\lambda}_{\lambda}$$

.

We define the following quantities

$$S_{\mu\nu} = T_{\mu\nu} - g_{\mu\nu}T^{\lambda}_{\lambda} \tag{4.20}$$

and

$$\Lambda = \tilde{\Lambda} - \frac{R}{2}$$

The equations (4.19) are called general equations because they apply to all physical scales and not only to gravity and the constant Λ is called the cosmological constant.

If $\tilde{\Lambda} = \frac{R}{2}$ then we obtain the Einstein's equations with $\Lambda = 0$, that is,

$$R_{\mu\nu} = kS_{\mu\nu} \tag{4.21}$$

If $\tilde{\Lambda} > \frac{R}{2}$ then we obtain the general equations with $\Lambda \neq 0$, that is,

$$R_{\mu\nu} = kS_{\mu\nu} + g_{\mu\nu}\Lambda \tag{4.22}$$

Equation (4.19) was derived from a member of the Planck patch, that is, from a conic section, so by virtue of self-similarity it would also express the evolution equation of the Planck patch at each stage of its evolution.

5. Conclusions

In this work the geometric definition of mass and the production of the equations of General Relativity are achieved through a one-to-one mapping of relations that determine the geometric shape expressing a flat two-dimensional space into a curved space. Through this process we do not need to invoke any principle of equivalence to find equations in curved spaces but to study the plane two-dimensional space and transfer relations from it to a curved space. This process is general and was applied in the case of foundation of equations of Maxwell (Krikos 2018). Through this process the flat two-dimensional boundary of the Universe can be considered as the common place of development of all the theories of Physics where through a holographic representation they are transferred to the three-dimensional curved space that we realize. This view can unify General Relativity and Quantum Mechanics since both theories are defined in the border of Universe.

6. References:

Carroll S, Spacetime and Geometry, Addison Wesley 2004

Gugggenheimer H, Differential Geometry,Dover Publications 1963

William Schulz and Alexia Schulz, A Practical Introduction to Differential Forms, 2012

Krikos Constantinos, Geometric Foundation and unification of the theories of physics, Hindustan Publishing Corporation, 2019

Krikos Constantinos, Foundation of Electromagnetic Theory, Open Science Journal 2018

Willmore T. An introduction to differential geometry, Dover Publications, 2012

Pathria R K, The theory of relativity, Dover Publications, 2003