OSJ | OPEN SCIENCE JOURNAL

RESEARCH ARTICLE

The Calibration of Hailpads upon the Greek National Suppression Program, using the Classical and Inverse Regression Methods

Kyriakos G. Tsitouridis[1]*

[1]Meteorological Applications Centre, Thessaloniki, Greece

*Corresponding author: Kyriakos G. Tsitouridis: k.tsitouridis@elga.gr

OPEN ACCESS

Citation: Tsitouridis K.G. (2019) The Calibration of Hailpads upon the Greek National Suppression Program, using the Classical and Inverse Regression Methods. Open Science Journal 4(1)

Received: 28th October 2018

Accepted: 18th September 2019

Published: 16th October 2019

Funding: The author(s) received no specific funding for this work

Competing Interests: The author have declared that no competing interests exists.

Abstract:

The hailpad, constructed from a plate of Styrofoam, is a simple instrument for recording hailfall. In addition to simply recording the hailfall, calibration of the instrument is required to obtain quantitative measurements of the hail. The calibration is a process leading to a calibration equation, a polynomial establishing a relationship between the diameter of a hailstone and the dent the hailstone is left on the surface of the hailpad. A hailpad network, consisted of 154 instruments, has established in Greece, in the context of the Greek National Hail Suppression Program operating for the protection of the agricultural cultivations from hail damage. For the calibration of the haipads of the network the well known "Energy Matching technique" has adopted and the Inverse Regression method is applied from the beginning of the operation of the network on 1984, for the obtainment of the calibration equation. In the present study, along with the Inverse Regression method hitherto applied, the Classical Regression method is examined and presented and, for the first time, inferential statistics are also introduced in order to establish a more stringent statistical procedure for the calibration of the hailpads. After the theoretical analysis of the two methods, the data from a calibration experiment were analyzed, calibration models obtained using both methods of regression, hail diameters were predicted with the two models and the results compared to each other. The comparison of the predictions with the two models shows that the very small differences that appear can be ignored, taking into account and the natural variability of the properties of the hail, so classical regression does not have any advantage over reverse regression. It is therefore shown that reverse regression, which is also used in other networks internationally, is quite satisfactory and as simple as its implementation, it is proposed as the main calibration method. Inferential statistics is suggested to be applied and prediction bands to be calculated in each future calibration experiment, giving greater validity to the results. In addition to this, the classical regression method can be used for continuous comparison of the results between the two methods.

Keywords: Hailpad, calibration, regression, classical regression, inverse regression, prediction bands

Introduction

The hailpad is a simple yet effective meteorological instrument for the study of the hail falls. The instrument introduced in 1959 [1] and has since been used by many researchers, for the study of the characteristics and climatology of hail and for the evaluation of hail suppression programs [2-6].

The hailpad was initially introduced in the Greek National Hail Suppression Program – GNHSP - during the period 1984 – 1988 when a hailpad network was established in the context of the evaluation experiment of the program based on a crossover design. After the end of the evaluation experiment, the hail suppression program continued as purely operational for the protection of the agricultural cultivations from hail damage and the Meteorological Applications Centre, a branch of the Hellenic Agricultural Insurance Organization-ELGA, is responsible for the operation of the Program. For the daily evaluation of the Program the hailpad network consisted of 154 instruments was maintained in operation till now.

There are few variations of the instrument [5, 7]. The variation of the instrument used in the GNHSP, consists of a pad of Styrofoam mounted horizontally in a case at the tope of a pole, about 1.5m above the ground, so that the upper surface exposed to the hail has dimensions of 27cmX27cm. The Styrofoam variation with the smooth skin is used, which is painted with a water mixture of white wall painting emulsion for protection against the sun radiation.

The response of the instrument is a dent formed in the surface of the pad, after the impact of a hail stone. From the dent that causes the hail stone on the surface of the hailpad, the researcher has to draw conclusions about the characteristics of the hail, so the calibration of the instrument is an essential stage for the measurement of certain characteristics of the hailstone. The calibration is a group of procedures that lead to the establishment of a mathematical relationship between the calibration standards used – calibration steel spheres - and the responses of the instrument. This mathematical relationship is then used to transform the responses of the instrument - the minimum diameters of the dents left on the surface of the pad - to estimates of the diameters of the hail stones responsible for the formation of the dents. The calibration of the hailpads in the GNHSP is performed applying the “Energy-Matching - EM” technique, originated with Schleusener and Jennings [1].

The practice hitherto upon the GNHSP is the obtainment of the calibration equation using the Inverse Regression method as shown below, without considering whether the method used is the most appropriate and whether the calibration data meets the requirements of the regression theory. The purpose of this work is to seek answers to these questions, to examine the alternative Classical Regretion method and to set a more stringent statistical procedure for the calibration of the hailpads.

For the obtainment of the calibration equation two different methods of linear regression are presented, the hitherto used Inverse Regression method and the Classical Regression method as well. Finally the data of a calibration experiment are analyzed with the two methods and prediction bands for new observations are produced.

The remainder of the work is organized as follows:

In section 2, the operation of the hailpad network is presented, along with the theoretical basis, the experimental procedures and the statistical methods for the analysis of the experimental data. In section 3 an application example is presented, analyzing the data of a calibration experiment, using both methods of regression, prediction bands are derived for both and finally the results are compared, closing with conclusion and recommendations.

Materials and methods

The operation of the hailpad network

Upon the GNHSP, a network of hailpad stations is in operation from 1984 till now, consisted of 154 stations from 2008 till now, in an area of 2,200 square km, with a mean of 3.8X3.8km2 area corresponding to each hailpad. During the hail period from March to September each year, the network is under continuous service. Every hailpad is replaced by a new one in two different instances, a) if it is affected by a storm b) if it is exposed to the sun radiation for more than 30 to 35 days, time period during which it keeps it's initial properties [5].

The identification of the hailpads affected by a storm is carried out using the Thunderstorm Identification, Tracking, Analysis and Nowcasting – TITAN - system [8], a software ingesting radar data from the EEC C-band meteorological radar installed at Filyron, a village about 20 km north of Macedonia Airport of Thessaloniki, Greece. The hailpads affected by storms in a day, are replaced in the next day as soon as possible. A common problem of a hailpad network is the possibility a hailpad to be affected by different storms in a day, a problem faced effectively by the use of the TITAN software and the analysis of the storms.

The hit hailpads are painted with black ink by means of a printmaking cylinder, in order the boundaries of the dents to become clear, and then scanned so that the hailpad surface to be converted to a digital image, ready for the analysis. The analysis is performed using an application built on the ImageProPlus software.

The Calibration of the Hailpads - Theory and Practices

Two groups of assumptions are accepted for the calibration of the hailpads, the first group is related to the physical properties of the hail stone and the Styrofoam and the second one is related to the statistical theory behind the regression method. For the purposes of this paper, both groups of the assumptions are briefly quoted. For a deeper study of the assumptions the reader may have recourse to the cited literature.

For the calibration of the hailpads and the data reduction certain assumptions are accepted as stated in [1, 3, 5]. Those assumptions accepted upon the GNHSP too, are briefly the following: a. The hailstone has a spherical shape, b. The hailstones that impact with the ground are rigid, c. The density of all hailstones is ρ_h=0.89g/cm^3, d. The drag coefficient of the hailstones is C_d=0.6, e. A hailstone falls vertically onto a hailpad, with its terminal velocity and f. The hailstones hit the hailpad once.

These assumptions are based on a summary of the findings of the research and in no case represent any individual hail fall, as hail has a great variability in nature [9], but they are a good basis for assessing the characteristics of the hail falls. The alternative, more accurate, methods are based on very expensive equipment to collect the real hailstones [10].

The purpose of the calibration of the hailpad is to establish a mathematical relation between the minor axis of a dent and certain properties of the hail stone making the dent [5]. As already mentioned in the Introduction, in the GNHSP the "Energy-Matching - EM" calibration technique has been adopted, originated with Schleusener and Jennings [1].

The EM technique does depend on the assumption that "spheres of equal diameters create dents of equal minor axes when the spheres have equal impact kinetic energy". As reported on [5] "Lozowski et al (1978a) have shown experimentally that this assumption holds for spheres of ice (ρ=0.9 g/cm^3), glass (ρ=2.5 g/cm3) and steel (ρ=7.8 g/cm^3). Experiments carried out in the National Hail Research Experiment - NHRE with polypropylene spheres (ρ=0.9 g/cm^3) and teflon spheres (ρ=2.2 g/cm^3) confirmed this result", p1305. The theory of dent formation first proposed by Lozowski et al [3] and developed further by Long et al., [5, 7], also supports this assumption. Not any similar experiment carried out in the GNHSP, but the dent formation theory referred in the mentioned literature has been adopted.

For the calibration of the hailpads, a steel simulation sphere is dropped from the proper height, so that the kinetic energy of the simulation sphere at impact with the hailpad is equal to the kinetic energy of a hailstone with the same diameter, falling with its terminal velocity.

In Long et al., [5, 7] the mathematical expression (1) obtained for the height from which a simulation sphere must be dropped in still air onto a hailpad so that the impact kinetic energy of the sphere equals that of a spherical hailstone of the same diameter, falling with its terminal velocity.

$$H_s = -\frac{2 \bullet \rho_s}{3 \bullet \rho_{a2} C_{ds}} \bullet \ln\left(1 - \frac{\rho_h \bullet \rho_{a2} \bullet g_1 \bullet C_{ds} \bullet (\rho_h - \rho_{a1})}{\rho_s \bullet \rho_{a1} \bullet g_2 \bullet C_{dh} \bullet (\rho_s - \rho_{a2})}\right) \bullet D \qquad (1)$$

In (1) ρ_s, and ρ_h are the densities of steel and hail respectively, C_{ds}, C_{dh} are the drag coefficients (dimensionless) of the steel and the hail respectively and D is the diameter of the simulation sphere, ρa1 is the density of air and g_1 is the acceleration of gravity near the field where the data are collected, while ρ_{a2} is the density of air and g_2 is the acceleration of gravity in the calibration laboratory. Taking into account that the laboratory of the GNHSP is close to the hailpad network and there is not significant difference in the longitude, latitude and altitude between the hailpad locations and the laboratory location, and assuming still air, it is accepted that the air densities are the same for both locations: $\rho_{a1}=\rho_{a2}=\rho$. Furthermore the calculations shown that the involved gravitational accelerations in the field and the laboratory are equal: $g_1=g_2=g$. So from the equation (1) the following simplified expression (2) is obtained:

$$H_s = -\frac{2 \bullet \rho_s}{3 \bullet \rho_a C_{ds}} \bullet \ln\left(1 - \frac{\rho_h \bullet C_{ds} \bullet (\rho_h - \rho_a)}{\rho_s \bullet C_{dh} \bullet (\rho_s - \rho_a)}\right) \bullet D \tag{2}$$

Substituting in the above equation (2) the values of the involved constant parameters Density of air: ρ_a =0.00119g/cm^3=1.19kg/m^3, Density of (steel) sphere: ρ_s=7.78g/cm^3=7780kg/m^3, Density of hail: ρ_h=0.89g/cm^3=890kg/m^3, Drag coefficient of the steel spheres: C_{ds}=0.45 (dimensionless), Drag coefficient of hail: C_{dh}=0.60 (dimensionless), the simplified equation (3) results:

$$H_s = 95.418764 \bullet D \tag{3}$$

Substituting in the above simplified equation the diameters of the twelve (12) simulation spheres used in the GNHSP, the following twelve pairs of values of the Table 1 are obtained, where D_s is the diameter of the simulation sphere and H_s is the drop height, both expressed in mm.

Table 1. Drop heights of the simulation spheres

No	D_s (mm)	H_s (mm)	No	D_s (mm)	H_s (mm)	No	D_s (mm)	H_s (mm)
1	6.35	606	5	12.70	1212	9	22.23	2121
2	7.94	758	6	13.49	1287	10	25.40	2424
3	9.53	909	7	14.29	1364	11	31.75	3030
4	11.11	1060	8	18.26	1742	12	34.93	3333

The minor axes of the dents left on the surface of the hailpad by the steel spheres are measured using the ImageProPlus software, and the data are used to develop a linear model relating the diameter of the sphere to the minor axis of the dent, by means of the least squares method.

Till the year 2010 a calibration experiment carried out at the beginning of the hail period for the total consignment of the hailpad material. From the year 2011 a calibration experiment is carried out for every packet of Styrofoam consisted of 14 plates with dimensions of 60cmX220cm, to minimize the errors related to the painting and inking procedures and the variability of the properties of the material. It could be better to apply the individual calibration method which is promising smaller errors [12] but the transition to this method is very difficult for practical and economical reasons, so the packet calibration is a good alternative as there is no need for the replacement of the existing infrastructure.

Beyond the above change, some improvements have been made between 2007 and 2009 with regard to calibration procedures. Particularly, a hailpad laboratory has been developed and equipped with a calibration device, a new inking cylinder with dimensions appropriate to cover a hailpad with black ink in one revolution and an A3 calibrated scanner. In addition, the analysis of the hailpads and the data reduction is performed using an application built on the ImageProPlus software. With these improvements, the error sources were drastically reduced and the run time of the calibration experiments is limited. Furthermore a hailpad technician has been trained to carry out all the necessary work, the painting, inking and scanning included, in order to greatly reduce the human error.

Under the new laboratory conditions and practices, which guarantee fewer and smaller measurement errors, there is the necessary background for introducing more stringent statistical analyses of the calibration data, enriched with statistical inference procedures.

Summary of the statistical methods for the calibration.

According to the research and the experience, in the case of hailpads there is a causal relationship of the form Y=f(X), between the diameter X of a hailstone and the diameter Y of the dent left by the hailstone on the surface of the hailpad. The most common relationship used is the linear one of the form $Y=\beta_0+\beta_1X$, while two or higher degree polynomials are proposed [11]. The coefficients β_0 and β_1 are estimated by regression procedures as shown below in this paragraph.

Due to the possible packet-to- packet variability of the hailpad material, a calibration experiment is carried out for every new packet of Styrofoam plates from the year 2011. Each one of the twelve (12) simulation steel spheres of the Table 1 is released ten (10) times from the proper height to hit the surface of sample hailpads collected from a new packet consisted of 14 plates of Styrofoam. After the end of the experiment, every hit sample hailpad is inked and scanned. Using the ImageProPlus software, the minimum diameters of the dents are measured and a table of 120 pairs of values (X_i, Y_i) is formed, where X_i is the diameter Ds of the simulation steel sphere and Y_i is the minor axis D_d of the dent on the surface of the hailpad.

In the case of the hail, naturally, the diameter of the hailstone (and the diameter - D_s - of the steel simulation sphere as well) is the independent variable X, as the hailstone is the cause of the dent formed on the surface of the hailpad, and the diameter of the dent - D_d -is the dependent or response variable Y.

As stated in Weiss, Neel A, 2012, p. 670, [13] certain assumptions (conditions) are accepted for the regression model:

"1. There are constants β_0 and β_1 such that, for each value X of the predictor variable, the conditional mean of the response variable is $\beta_0+\beta_1X$.

2. The conditional standard deviations of the response variable are the same for all values of the predictor variable.

3. For each value of the predictor variable, the conditional distribution of the response variable is a normal distribution.

4. The observations of the response variable are independent of one another".

In the calibration of the hailpads, the diameter and the density of the simulation spheres are involved in the obtainment of equation (1) and they are measured with a very small error which can be ignored, while the measurement of the diameter of the dent formed on the surface of the Styrofoam is subject to errors, because of various parameters such as the differences of the properties of Styrofoam mass from point to point, the different thickness of the ink film, the tiny anomalies of the Styrofoam surface, the errors associated with the analysis software and so on. Also, the above assumptions 2 and 3 are supposed to be valid.

In the case of the calibration of hailpads, during the experimental procedure it is ensured that the replicate drops of a simulation sphere are absolutely independent of one another and care is taken to ensure that each dent is formed away from the previous one, so the assumption 4 is always valid.

A good approach to building a regression model so as not to violate the basic assumptions mentioned above is to consider the diameter of the steel as the independent variable and the diameter of the dent as the dependent variable as this happens naturally. This is the method of Classical Regression. However, the Inverse Regression method is widely used not only for the calibration of the hailpads but generally in the calibration of measuring devices. According to this method, the diameter of the dent is considered as the independent variable and the diameter of the steel ball is considered as the dependent variable.

The two methods examined and compared by many researchers. Krutchkoff, R.G.,[14] compared the two methods of linear calibration by Mode Carlo methods and concluded that the Inverse Regression is slightly better than the Classical one. Shucla, G.K.,[15] concluded that both methods have advantages depended on the sample size and suggested the classical estimators for general purposes and large sample sizes. Parker et al [16] made a similar comparison of the two methods and proposed prediction intervals for both.

In the case of the Inverse Regression for the calibration of the hailpads, the value of the minor axis of the dent, which is used as predictor, depends on various parameters, measured with not negligible errors, as mentioned above. So this method is expected to "violate the simple linear regression assumption that the predictor is measured with negligible error",[16].

In this paper, both, the Classical Regression and the Inverse Regression are presented. After the theoretical presentation of the two methods, the same data set from a hailpad calibration experiment is used to build a linear model and construct prediction intervals in the case of the Classical Regression and in the case of Inverse Regression.

In both methods, the same regression and statistical inference theory apply. In Montgomery et al [17], (Chapter 11), there is a comprehensive presentation of the Simple Linear Regression method, which is briefly presented in the following.

Let X to be the independent (predictor) variable and Y the dependent (response) variable. The values X_i of X are all measured with negligible error while the values of Y_i of Y are measured with random errors. Assuming that a simple linear model is appropriate, the true model is:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \tag{4}$$

In the model (4) β_0 is the Y-intercept, β_1 is the slope of the line, and the εi's are random errors. The random errors ε_i assumed to be independent and normally distributed with a mean of zero (0) and a variance of σ^2, $\varepsilon_i \sim N(0,\sigma^2)$.

The regression function (5) based on n observations is obtained, using the least squares method.

$$\hat{Y} = b_0 + b_1 \hat{X} \tag{5}$$

In the equation (5) b_0 and b_1 are unbiased point estimators of β_0 and β_1 respectively, given by the equations (6) and (7) [17].

$$b_1 = \frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)}{\sum_{i=1}^{n}(X_i - \bar{X})^2_{\;1}} = \frac{S_{XY}}{S_{XX}} \tag{6}$$

$$b_0 = \bar{Y} - b_1\bar{X} \tag{7}$$

After the equation (5) is obtained, a hypothesis testing is performed to examine if $\beta_1 \neq 0$ and $\beta_0 \neq 0$, using the test statistic $t^* = (b_i - \beta_i)/s\{b_i\}$, which follows the t-distribution, because the population statistics are unknown and only sample statistics are available.

For $(1-\alpha)100\%$ confidence interval the null hypothesis H_0: $\beta_1=0$ versus the alternative Ha: $\beta_1 \neq 0$ is examined. Setting $\beta_1=0$ the t* statistic becomes: $t^*=b_1/s\{b_1\}$. A two tailed test is performed and the null hypothesis H_0 is rejected and the alternative H_a is accepted if $|t^*|>t_{(1-\alpha/2;n-2)}$, where $t_{(1-\alpha/2;n-2)}$ is the critical value of the t-distribution of the two tailed test for α level of significance and n-2 degrees of freedom. The real meaning of $\beta_1\neq0$ is that the slope β_1 is significantly different from zero, or otherwise for an increase of the independent variable by 1.0, there is an increase in the dependent variable by β_1.

Following a similar procedure, a hypothesis test is performed about the intercept β_0. For $(1-\alpha)100\%$ confidence interval the null hypothesis H_0: $\beta_0=0$ versus the alternative H_a: $\beta_0\neq0$ is examined. Setting $\beta_0=0$ the t* statistic becomes: $t^*=b_0/s\{b_0\}$. The null hypothesis H_0 is rejected and the alternative H_a is accepted if $|t^*|>t_{(1-\alpha/2;n-2)}$.

Given the point estimators b_1 of β_1 and b_0 of β_0, $(1-\alpha)100\%$ confidence limits for β_1 and β_0 are:

$$b_1 \pm t_{(1-\frac{\alpha}{2};n-2)} \bullet s\{b_1\} \text{ and } b_0 \pm t_{(1-\frac{\alpha}{2};n-2)} \bullet s\{b_0\} \tag{8}$$

From the fitted model (5), for a given value x0 of the predictor variable X, the mean estimated value $\hat{y}_0$ of the response variable is

$$\hat{y}_0 = b_0 + b_1 x_0 \tag{9}$$

A $(1-\alpha)100\%$ confidence interval for $\hat{y}_0$ is given by the expression (10).

$$\hat{y}_0 - t_{(1-\frac{\alpha}{2};n-2)} \bullet \sqrt{s^2\left[\frac{1}{n} + \frac{(x_0 - \bar{X})^2}{S_{xx}}\right]} \leq \hat{y}_0 \leq \hat{y}_0 + t_{(1-\frac{\alpha}{2};n-2)} \bullet \sqrt{s^2\left[\frac{1}{n} + \frac{(x_0 - \bar{X})^2}{S_{xx}}\right]} \tag{10}$$

The above confidence interval is minimized at $x_0 = \bar{X}$ and is widening as $|x_0 - \bar{X}|$ increases.

Another way to judge the adequacy of the regression model is to calculate *the coefficient of determination* $R^2 = SS_R/SS_T = \sum_{i=1}^{N}(\hat{Y}_i - \bar{Y})^2 \Big/ \sum_{i=1}^{N}(Y_i - \bar{Y})^2$ (11)

Or $R^2 = SS_R / SS_T = 1 - SS_E / SS_T = 1 - \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2 \Big/ \sum_{i=1}^{N} (Y_i - \bar{Y})^2$ (12)

As closer to 1 is the value of the R_2 as stronger the correlation between the two parameters is.

The Classical Regression method.

In the case of the Classical Regression method also known as Inverse Prediction, when a new observation Y_0 of a dent becomes available, the point estimation of the level X_0, the diameter of a hailstone, that gave rise to this new observation is made by inverting the model (5), so that given Y0 and $b_1 \neq 0$, a point estimator of X_0 is: $\hat{X}_0 = \frac{Y_0 - b_0}{b_1}$ (13)

$$\hat{X}_0 - t_{(1-\frac{\alpha}{2};n-2)} \bullet s\{predX\} \leq \hat{X}_0 \leq \hat{X}_0 + t_{(1-\frac{\alpha}{2};n-2)} \bullet s\{predX\} \quad (14)$$

In (23) $s\{predX\} = \sqrt{s^2\{predX\}}$ and $s^2\{predX\} = \frac{s^2}{b_1^2} \bullet \left[1 + 1/n + \left(\hat{X}_0 - \bar{X}\right)^2 / S_{xx}\right]$ (15)

and the $t_{(1-\frac{\alpha}{2};n-2)}$ is the critical value of the t-distribution of the two tailed test for α level of significance and n-2 degrees of freedom.

As can be seen in (15), the above prediction interval of $\hat{X}_0$ is minimized at $\hat{X}_0 = \bar{X}$ and is widening as $|\hat{X}_0 - \bar{X}|$ increases.

The Inverse Regression method.

In the case of the Inverse Regression method, as stated by [16], [18] (pp 58-59), the variable X, the diameter of the simulation sphere, is treated as the response (dependent) variable and the Y, the minimum diameter of the created dent, is treated as the predictor (independent) variable. Then the model (4) can be written in the form (16).

$$X_i = \gamma_0 + \gamma_1 Y_i + \varepsilon_i' \quad (16)$$

In the model (16) γ_0 is the X-intercept (ordinate), γ_1 is the slope of the line and ε_i' are random errors. The random errors ε_i' assumed to be independent and identically distributed as normal with a mean of zero (0) and a variance of σ2, ε_i' ~N(0,σ2).

The regression function (17) based on n observations is obtained, using the least squares method.

$$\hat{X} = c_0 + c_1 \hat{Y} \quad (17)$$

In the equation (17) c_0 and c_1 are unbiased point estimators of γ_0 and γ_1 respectively, given by the equations (18) and (19):

$$c_1 = \frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2} = \frac{S_{XY}}{S_{YY}} \tag{18}$$

$$c_0 = \bar{X} - c_1\bar{Y} \tag{19}$$

$$S_{YY} = \sum_{i=1}^{n}(Y_i - \bar{Y})^2 \tag{20}$$

An unbiased estimator of σ^2 for the regression line (17) is given in equation (21).

$$s_I^2 = MSE = \frac{SS_E}{n-2} = \frac{1}{n-2}\sum_{i=1}^{n}(X_i - \hat{X}_i)^2 \tag{21}$$

After the model is built, the null hypothesis H_0: $c_i=0$ versus the alternative H_a: $c_i \neq 0$ is examined, as in paragraph 2.3.1, using the test statistic $t^* = (c_i - \gamma_i)/s\{c_i\}$.

For (1-α)100% confidence interval the null hypothesis H_0: $c_i=0$ versus the alternative H_a: $c_i \neq 0$ is examined. Setting $\gamma_1=0$ the t* statistic becomes $t^* = c_i/s\{c_i\}$.

Given the point estimators c_1 of γ_1, c_0 of γ_0 and s^2 of σ^2, (1-α)100% confidence limits for γ_1 and γ_0 are given by equations (22) and (23).

$$c_1 \pm t_{(1-\frac{\alpha}{2};n-2)} \bullet s\{c_1\} \tag{22}$$

$$c_0 \pm t_{(1-\frac{\alpha}{2};n-2)} \bullet s\{c_0\} \tag{23}$$

In equations (22) and (23) $s\{c_1\} = \sqrt{s^2\{c_1\}} = \sqrt{s^2 / S_{YY}}$, $s\{c_0\} = \sqrt{s^2\{c_0\}} = \sqrt{s^2 \bullet \left[1/n + \bar{Y}^2 / S_{YY}\right]}$.

As stated in [16], [17] (pp 392), given a new or future observation Y_0, a (1-α)100% prediction interval for the estimated value $\hat{X}_0 = c_0 + c_1 Y_0$ is presented in expression (24):

$$\hat{X}_0 \pm t_{(1-\frac{\alpha}{2};n-2)} \bullet s\{pred\} \tag{24}$$

In (24) $s\{pred\} = \sqrt{s_I^2 \bullet [1 + 1/n + \left(Y_0 - \bar{Y}\right)^2 / s_{YY}]}$ (25)

In (25) the s_I^2 given in equation (21) is used. Note that Y_0 is a random variable measured with not negligible error, so is expected to violate the fundamental regression assumption which states that the regressor is measured with negligible error.

Results and Discussion

In this section a set of a calibration experiment data is analyzed with both methods presented above and the results are compared to each other.

Data set

During the hail period of 2017, five (5) packets of Styrofoam plates have calibrated and the calibration equations obtained for each one, using the inverse regression method as usual. The calibration data of the fourth Styrofoam packet will be used in the following to derive the calibration equation and a 95% prediction band running both methods. The calibration data are included in the following Table 2.

Table 2. Calibration data of the fourth packet of Styrofoam

X	Y	X	Y	X	Y	X	Y	X	Y	X	Y
6.35	3.001	7.94	4.396	9.53	5.710	11.11	7.005	12.70	9.163	13.49	10.183
6.35	2.695	7.94	4.033	9.53	6.255	11.11	7.311	12.70	9.496	13.49	10.397
6.35	2.856	7.94	4.234	9.53	6.227	11.11	7.069	12.70	9.103	13.49	9.991
6.35	2.424	7.94	4.204	9.53	5.695	11.11	7.220	12.70	9.060	13.49	10.107
6.35	3.170	7.94	3.943	9.53	5.808	11.11	7.191	12.70	8.916	13.49	9.513
6.35	2.935	7.94	4.204	9.53	5.760	11.11	7.313	12.70	8.417	13.49	9.794
6.35	3.035	7.94	4.462	9.53	6.181	11.11	7.921	12.70	9.141	13.49	9.642
6.35	2.935	7.94	4.526	9.53	5.869	11.11	7.397	12.70	9.066	13.49	9.865
6.35	2.919	7.94	4.317	9.53	5.896	11.11	7.705	12.70	9.065	13.49	9.981
6.35	3.007	7.94	3.940	9.53	5.927	11.11	7.604	12.70	8.983	13.49	10.287

X	Y	X	Y	X	Y	X	Y	X	Y	X	Y
14.29	10.754	18.26	15.825	22.23	20.543	25.40	23.597	31.75	30.462	34.93	33.443
14.29	10.583	18.26	15.870	22.23	20.137	25.40	23.805	31.75	30.186	34.93	34.083
14.29	10.397	18.26	15.715	22.23	20.290	25.40	23.758	31.75	30.381	34.93	33.268
14.29	10.494	18.26	15.713	22.23	20.151	25.40	23.572	31.75	30.480	34.93	32.941
14.29	10.178	18.26	15.698	22.23	20.458	25.40	24.207	31.75	30.995	34.93	34.378
14.29	10.061	18.26	15.485	22.23	20.913	25.40	24.490	31.75	30.583	34.93	33.951
14.29	10.716	18.26	15.579	22.23	20.796	25.40	23.883	31.75	30.669	34.93	33.885
14.29	10.597	18.26	15.488	22.23	21.163	25.40	24.375	31.75	30.252	34.93	34.053
14.29	10.502	18.26	15.316	22.23	20.670	25.40	24.294	31.75	30.565	34.93	33.978
14.29	10.620	18.26	15.538	22.23	20.493	25.40	23.936	31.75	30.308	34.93	34.130

In the above Table 2, X is the diameter of the simulation sphere expressed in mm and Y is the minimum diameter of the dent left on the surface of the hailpad, expressed in mm too. For each one diameter of the simulation spheres, there are ten (10) values of the minimum dent diameter. The total number of data pairs is n=120.

Data analysis with the Classical Regression method

Let X to be treated as the independent (predictor) variable and Y to be treated as the dependent (response) variable. The values X_i of X are all measured with negligible error while the values Y_i of Y are measured with random errors. Assuming that a simple linear model is appropriate, the true model is: $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$, where β_0 is the Y-intercept (ordinate), β_1 is the slope of the line, and the ε_i's are random errors. The random errors ε_i assumed to be independent and identically distributed as normal with a mean of zero (0) and a variance of σ^2, $\varepsilon_i \sim N(0,\sigma^2)$.

Applying the least squares method of the paragraph 2.3 with the data of the Table 2, the regression model $\hat{Y} = b_0 + b_1 \hat{X}$ is obtained, where b_0=-4.73033688 and b_1=1.11195779 are unbiased point estimators of β_0 and β_1, DF=118, the variance s^2=0.2134268, the standard deviation s=0.46198138, the coefficient of determination R^2=0.99788034. In the following Table 3 there is a summary of the values of the b_0 and the b_1 parameters.

Table 3. Parameters b_0, b_1

Parameter	Value	95% confidence limits	St. deviation (s)	t*
b0	-4.73033688	(-4.912471921 , -4.54820184)	0.09197475	-51.4308218
b1	1.11195779	(1.102615228 , 1.121300345)	0.00471782	235.69338

For n-2=118 degrees of freedom, α=0.05 level of significance and two tailed Hypothesis testing for the parameters b_1 and b_0 the t-value is $t_{(0.975,118)}$=1.98027224.

The Hypothesis testing has as in the following:

Hypothesis testing for β_1

Null Hypothesis H_0: $\beta_1=0$

Alternative Hypothesis H_a: $\beta_1 \neq 0$

Because $|t^*|$=235.69338> $t_{(0.975,118)}$=1.98027224 the Null Hypothesis is rejected and the Alternative hypothesis is accepted, so the slope β_1 is not zero, which means that there is a statistically significant linear relation of the minimum dent diameter to the hail diameter, at 95% level of confidence.

Hypothesis testing for β_0

Null Hypothesis H0: $\beta_0=0$

Alternative Hypothesis H_a: $\beta_0 \neq 0$.

Because $|t^*|$=51.4308218> $t_{(0.975,118)}$=1.98027224, the Null Hypothesis is rejected and the Alternative Hypothesis is accepted, which states that the Y-intercept is not zero, at 95% level of confidence.

When a new observation of a dent with minimum diameter Y_0 becomes available, from the model $\hat{Y} = -4.73033688 + 1.11195779 \cdot \hat{X}$, using the equation (13), the mean estimated value X_0 of the hail diameter is obtained.

A 95% Prediction Interval for the estimated hail diameter X_0 is calculated using the equation (14). Calculations performed for minimum dent diameters from 2mm to 35mm.

Data analysis with the Inverse Regression method

In the case of the Inverse Regression method, the variable X, the diameter of the simulation sphere, is treated as the response (dependent) variable and the variable Y, the minimum diameter of the created dent, is treated as the predictor (independent) variable. Assuming that a simple linear model is appropriate, the true model is: $X_i = \gamma_0 + \gamma_1 Y_i + \varepsilon_i^{'}$, where γ_0 is the X-intercept (ordinate), γ_1 is the slope of the line and $\varepsilon_i^{'}$ are random errors. The random errors $\varepsilon_i^{'}$ assumed to be independent and identically distributed as normal with a mean of zero (0) and a variance of σ^2, $\varepsilon_i^{'} \sim N(0,\sigma^2)$.

Applying the Least Squares method of the paragraph 2.3 on the data of the Table 2, the regression model $\hat{X} = c_0 + c_1 \hat{Y}$ is obtained, where c_0=4.28176748 and c_1=0.89740848 are unbiased point estimators of γ_0 and γ_1 respectively, DF=118, variance s^2=0.17224666, standard deviation s=0.4150261 and the coefficient of determination R^2=0.99788034. In the following Table 4 there is a summary of the values of the c_0 and the c_1 parameters.

Table 4. Values of the parameters c_0 and c_1

Parameter	Value	95% confidence limits	St. deviation (s)	t*
c_0	4.28176748	(4.148957984 , 4.414576973)	0.06706628	63.8438185
c_1	0.89740848	(0.88986854 , 0.904948412)	0.00380753	235.69338

For n-2=118 degrees of freedom, α=0.05 level of significance and two tailed testing $t_{(0.975,118)}$=1.98027224, the Hypothesis testing has as in the following:

Hypothesis testing for γ_1

Null Hypothesis H_0: $\gamma_1=0$

Alternative Hypothesis H_a: $\gamma_1 \neq 0$

Because $|t^*|$=235.69338> $t_{(0.975,118)}$=1.98027224 the Null Hypothesis is rejected and the Alternative Hypothesis is accepted, so the slope γ_1 is not zero, which means that there is a statistically significant linear relation of the hail diameter to the minimum dent diameter, at 95% level of confidence.

Hypothesis testing for γ_0

Null Hypothesis H_0: $\gamma_0=0$

Alternative Hypothesis Ha: $\gamma_0 \neq 0$

Because $|t^*|$=63.8438185> $t_{(0.975,118)}$=1.98027224, the Null Hypothesis is rejected and the Alternative Hypothesis is accepted, which states that the X-intercept (ordinate) γ_0 is not zero, at 95% level of confidence.

When a new observation of a dent with minimum diameter Y_0 becomes available, the mean estimated value X_0 of the hail diameter is obtained based on the model $\hat{X} = 4.28176748 + 0.89740848 \cdot \hat{Y}$ and a 95% Prediction Interval for the hail diameter X_0 is calculated using the equations (24) and (25). Calculations performed for minimum dent diameters from 2mm to 35mm.

Comparison of the results of the two models and discussion

In the following Table 5, the prediction values and the 95% prediction intervals of the calculations performed in the paragraphs 3.2 and 3.3 are presented.

Table 5. Comparison of the results of the Classical and Inverse regression methods

Y_o	X_{o_cl}	CL_l	CL_{up}	X_{o_In}	In_l	In_{up}	$\Delta X_o(Cl\text{-}In)$	ΔX_{o_l}	ΔX_{o_up}
2	6.053	5.221	6.884	6.077	5.246	6.907	-0.024	-0.025	-0.023
3	6.952	6.121	7.783	6.974	6.144	7.804	-0.022	-0.023	-0.021
4	7.851	7.021	8.681	7.871	7.042	8.700	-0.020	-0.021	-0.019
5	8.751	7.921	9.580	8.769	7.940	9.597	-0.018	-0.019	-0.017
6	9.650	8.821	10.479	9.666	8.838	10.494	-0.016	-0.017	-0.015
7	10.549	9.721	11.377	10.564	9.736	11.391	-0.014	-0.015	-0.013
8	11.449	10.621	12.276	11.461	10.634	12.288	-0.012	-0.013	-0.012
9	12.348	11.521	13.175	12.358	11.532	13.185	-0.011	-0.011	-0.010
10	13.247	12.420	14.074	13.256	12.430	14.082	-0.009	-0.010	-0.008
11	14.147	13.320	14.973	14.153	13.328	14.979	-0.007	-0.008	-0.006
12	15.046	14.219	15.872	15.051	14.225	15.876	-0.005	-0.006	-0.004
13	15.945	15.119	16.771	15.948	15.123	16.773	-0.003	-0.004	-0.002
14	16.844	16.018	17.671	16.845	16.020	17.671	-0.001	-0.002	0.000
15	17.744	16.918	18.570	17.743	16.918	18.568	0.001	0.000	0.002
16	18.643	17.817	19.469	18.640	17.815	19.466	0.003	0.002	0.004
17	19.542	18.716	20.369	19.538	18.712	20.363	0.005	0.004	0.006
18	20.442	19.615	21.268	20.435	19.609	21.261	0.007	0.006	0.007
19	21.341	20.514	22.168	21.333	20.507	22.158	0.009	0.008	0.009
20	22.240	21.413	23.068	22.230	21.404	23.056	0.010	0.010	0.011
21	23.140	22.312	23.967	23.127	22.301	23.954	0.012	0.011	0.013
22	24.039	23.211	24.867	24.025	23.198	24.852	0.014	0.013	0.015
23	24.938	24.110	25.767	24.922	24.094	25.750	0.016	0.015	0.017
24	25.838	25.008	26.667	25.820	24.991	26.648	0.018	0.017	0.019
25	26.737	25.907	27.567	26.717	25.888	27.546	0.020	0.019	0.021
26	27.636	26.806	28.467	27.614	26.785	28.444	0.022	0.021	0.023
27	28.536	27.704	29.367	28.512	27.681	29.342	0.024	0.023	0.025
28	29.435	28.602	30.267	29.409	28.578	30.241	0.026	0.025	0.027
29	30.334	29.501	31.168	30.307	29.474	31.139	0.028	0.027	0.028
30	31.234	30.399	32.068	31.204	30.371	32.038	0.029	0.029	0.030
31	32.133	31.297	32.968	32.101	31.267	32.936	0.031	0.030	0.032
32	33.032	32.196	33.869	32.999	32.163	33.835	0.033	0.032	0.034
33	33.931	33.094	34.769	33.896	33.059	34.733	0.035	0.034	0.036
34	34.831	33.992	35.670	34.794	33.955	35.632	0.037	0.036	0.038
35	35.730	34.890	36.571	35.691	34.851	36.531	0.039	0.038	0.040

In the Table 5, given a new observation $\mathbf{Y_0}$, the mean predicted value of the hail diameter using the classical regression method is $\mathbf{X_{o\_cl}}$, the lower prediction limit is $\mathbf{CL_l}$ and the upper prediction limit is $\mathbf{CL_{up}}$, and the mean predicted value of the hail diameter using the inverse regression method is $\mathbf{X_{o\_In}}$, the lower prediction limit is $\mathbf{In_l}$ and the upper prediction limit is $\mathbf{In_{up}}$, $\mathbf{\Delta X_o(Cl\text{-}In)}$ is the difference of the predicted values between the classical and the inverse method, and ΔX_{o_l} and ΔX_{o_up} are the differences between the two methods for the lower and upper prediction limits respectively.

Looking at the values of the Table 5 it can be seen that the differences between the mean estimated values X_0 of the hail diameter with the two methods are very small and any difference appears in the second decimal. Given that the unit of measurement is the millimetre (mm), the calculated differences of the Table 5 in the predicted values with the two regression methods can be ignored, compared to the errors coming from the large variability of the natural hail [9] and other sources of errors [12].

It can also be seen in the Table 5 that for values of Y_0 from 2mm to up to 14mm the average estimated value X_0 with the Classical regression method is less than that with the Inverse regression method, while for values greater than 15mm the estimated values with the Classical method are larger than that estimated with the Inverse method. The lower and upper limits of 95% confidence intervals show the same behaviour. The slightly greater slope of the regression line of the Classical method is expected mathematically and does not change the general conclusion that the results are the same.

Concluding remarks and recommendations

In this paper two statistical methods for the calibration of the hailpads examined, calibration using the Classic Regression method and calibration using the Inverse Regression method. Some principles of statistical inference were also applied to examine the extent to which the results are statistically acceptable. Finally, the two methods were applied to the data of a calibration experiment taken using the Energy Matching technique and the results were compared to each other. The comparison shows that the results are similar.

While the Classic Regression method is preferable from a statistical point of view, since it takes the diameter of the hail as the independent variable, the Inverse Regression method gives almost the same results and it is easier, not so much to obtain the calibration equation but mainly for the calculation of the prediction bands. It is therefore not advisable to replace the Inverse Regression method used so far in the GNHSP with the Classical Regression one, but it is recommended both methods to be used, along with the introduction of the statistical inference procedures presented in this work.

The analysis of the hit hailpads of the Greek network of the years 2008 to 2017 shows that the diameter of the hail rarely exceeds 26mm, so it is recommended the calibration range to extend till this limit and the calibration experiment to performed using a new set of calibration steel spheres having diameters of the metric system from 6mm to 26mm, increasing by two (2) mm. If some rare hail fall will occur with diameters of hailstones greater than 26mm, then a special calibration can be performed, using calibration spheres with diameters greater than 26mm. In such a case second order polynomials can also be tried for better fitting of the model.

Acknowledgements

I would like to thank Dr José Luis Sanchez, Professor of Applied Physics, University of León, Spain, for the hospitality he offered me at his Laboratory and the fruitful discussions with him and his collaborators about the calibration of the hailpads, especially in the topic of the good laboratory practices to minimize the error sources, which helped me to proceed to some improvements in the calibration procedures at the hailpad laboratory of ELGA. I would like to acknowledge my collaborators Mr. Tegoulias Ioannis, Physicist – Meteorologist, for his involvement in the calibration procedure, and Mr. Amarantides Konstantinos, hailpad technician, who carries out a large part of the calibration work. In conclusion, I would like to thank the anonymous reviewer of the present work, whose helpful comments and suggestions were valuable to me.

References:

1. Schleusener,R.A., and P.C.Jennings, 1960: An energy method for relative estimates of hail intensity. Bull. Amer. Meteor. Soc., 41, 372-376.
2. Long, Alexis B., 1978: Design of Hail Measurement Networks. Atmosphere-Ocean, 16, 35-48
3. Lozowski,E.P., and G.S.Strong, 1978a: On the Calibration of Hailpads. J.Appl.Meteor., 17, 521-528.
4. Mezeix, Jean-Francois, P. Admirat, 1978: The measurement of Hail at Ground Level. Atmosphere-Ocean, 16, 61-68
5. Long,A.B., R.J.Matson and E.L.Crow, 1980: The hailpad: Materials, Data Reduction and Calibration. J.Appl.Meteor., 19, 1300-1313
6. Sanchez, J.L., B. Gil-Robles, J. Dessens, E. Martin, L. Lopez, J.L. Marcos, C. Berthet, J.L. Fernadez, E. Garcia-Ortega, 2009: Characterization of hailstone size spectra in hailpad networks in France, Spain and Argentina. Atmospheric Research 93 (1-3), 641-654
7. Long, Alexis B., Richard J. Matson and Edwin L. Crow, 1979: The hailpad: Construction and Materials, Data Reduction and Calibration. NCAR TECHNICAL NOTE (NCAR/TN-144+STR)
8. Dixon M. and G. Wiener, 1993: "TITAN: Thunderstorm Identification, Tracking, Analysis, and Nowcasting – a radar based methodology", J. Atmos. Ocean. Technol., vol. 10, 6 pp. 785-797. Software available at: http://www.rap.ucar.edu/projects/titan/home/index.php
9. Macklin, W.C.: The Characteristics of Natural Hailstones and Their Interpretation. Paper In: Hail: A Review of Hail Science and Hail Suppression. Edited by Foot, G. Brant and Charles A. Knight. Meteorological Monographs, Volume 16, American Meteorological Society, Boston 1977.
10. Matson, Richard J. and Arlen W. Huggins, 1980: The Direct Measurement of the Sizes, Shapes and Kinematics of Falling Hailstones. Journal of The Atmpspheric Sciences, 37, 1107-1125.
11. Lozowski,E.P., and G.S.Strong, 1978b: Further reflections on the Calibration of Hailpads. Atmosphere-Ocean, 16, 69-80.
12. Palentia, Covadonga, Claude Berthet, Marta Massot, Amaya Castro, Jean Dessens, Roberto Fraile, 2007: On the individual calibration of hailpads, Atmospheric Research 83, 493-504
13. Weiss, Neel A. Introductory Statistics, 9th Edition, 2012, Addison-Wesley.
14. Krutchkoff, R.G., (1967). Classical and Inverse Reggression Methods of Calibration. Technometrics. 9, No 3, 425439.
15. Shucla, G.K., 1972: On the Problem of Calibration. Technometrics. 40, No 3, 547-553.
16. Parker, Peter A.; Vining, G.Geofrey; Wilson, Sara R.; Szarka II, John L.; Johnson, Nels G., 2010: The Prediction Properties of Classical and Inverse Reggression for The Simple Linear Calibration Problem. Journal of Quality Technology, 42, No. 4, 332-347
17. Montgomery, Douglas C. and George C. Runger. Applied Statistics and Probability for Engineers. Third Edition 2003. John Wiley & Sons, Inc,
18. Kutner, M.H.; Nachtsheim, C.J.; Neter J.; and Li W. (2005). Applied Linear Statistical Models. 5th ed., NY: McGraw Hill/Irwin.