OSJ OPEN SCIENCE JOURNAL

RESEARCH ARTICLE

Foundation of Electromagnetic Theory

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OPEN ACCESS

Citation: Krikos C. (2018) Foundation of Electromagnetic Theory. Open Science Journal 3(1)

Received: 1st November 2017

Accepted: 23rd January 2018

Published: 19th February 2018

Funding: The author(s) received no specific funding for this work

Competing Interests: The author have declared that no competing interests exists.

Abstract:

In this paper equations in R^3 which are illustrations of "linear" ellipses, i.e. ellipses which tend to become segments of a geodesic of R^2, because their eccentricities tend to unit $(\varepsilon \to 1^-)$ will be found. During a linearization process of ellipses, varying vectors will be mapped, from which ellipses and their relations in R^2 , to varying vector fields and their relations in R^3 are defined. These vector fields and their relations in R^3 are called "**holographic**". At the limit $\varepsilon \to 1$, the holographic relationships are formalistically similar to Maxwell's equations. This is a theoretical derivation of Maxwell's equations and not a systematic classification of experimental data as Maxwell did.

Keywords: Shadow-generators, Conics, Holographic relations, Wave quanta

Introduction

Ancient Greek mathematicians and astronomers for any problem that emerged in their observation space, i.e. in three-dimensional Euclidean R^3 space and was not immediately solvable, proceed along the following method to solve it:

They were trying to formulate a problem of R^3 to a problem of the two-dimensional Euclidean space R^2, solve it there and then transfer its solution to R^3.

Through this method many problems were solved, as the Delian problem, the measurement of the distance from Earth to the Moon and from the Sun to the

Moon by Aristarchus of Samos, the measurement of the radius of the Earth by Eratosthenes, etc.

We shall apply the same method for conic sections, which are plane geometrical objects, because they display the same general features to physical entities (bodies and waves) in R^3, i.e. closed conics have as general features those of *localization* and *non-divisibility* as bodies and open conics have as general features those of *extensibility* and *divisibility* (divisibility comes from the two branches of hyperbolas) *as* waves [1]. So, through one-to-one correspondences between geometric quantities of conics (scalars or vectors) and physical quantities (scalars or vector fields) of physical entities all physical theories in R^3 are developed. We shall call conics as "**shadow-generators**" of physical entities.

During a linearization process of ellipses, varying vectors will be mapped, from which ellipses and their relations in R^2, to varying vector fields and their relations in R^3 are defined.

Furthermore, from the holographic relations for $\varepsilon = 1$, equations result which are formalistically similar to homogeneous and non-homogeneous wave equations. The shadow-generator of each wave equation in R^3, is the equation of a parabola in R^2, which is expressed either with respect to a coordinate system with origin at its focus (corresponded to a homogeneous wave equation), or to a coordinate system with origin at any point of its symmetry axis between its focus and its directrix (corresponded to a non-homogeneous wave equation). In the second case, the equation of this parabola is formalistically similar to the equation of an equilateral hyperbola.

Each constant conic is constructed by the constant vectors $\boldsymbol{\mathcal{L}_c}, \boldsymbol{\mathcal{R}_c}$, which have expressions that are given by the following relations.

$$\boldsymbol{\mathcal{L}_c} = \boldsymbol{r}_c(s)\times\boldsymbol{u}_c(s) \tag{1.1}$$

$$\boldsymbol{\mathcal{R}_c} = \boldsymbol{u}_c(s)\times\boldsymbol{\mathcal{L}_c} - \boldsymbol{r}_c(s)\left(\frac{k_c}{r_c(s)}\right) \tag{1.2}$$

where $\boldsymbol{r}_c(s), \boldsymbol{u}_c(s)$ are the position vector and the tangent vector of a point of a conic section respectively, k_c is a constant that characterizes a given conic and $r_c(s)$ is the measure of $\boldsymbol{r_c(s)}$. Each point of a conic section is characterized by its arc length s measured from an arbitrary origin on it.

Vector $\boldsymbol{\mathcal{L}_c}$ is perpendicular to the face of the plane on which a conic section is lying, having a direction from this face outwards and $\boldsymbol{\mathcal{R}_c}$ (**Rünge-Lentz vector**) is a vector lying on the plane of conic section. If the dot product of the vector $\boldsymbol{\mathcal{R}_c}$ and the unit vector $\boldsymbol{e_r} = \frac{\boldsymbol{r}_c(s)}{r_c(s)}$ is taken, the relation (1.2) becomes the equation of a conic section in polar coordinates in its plane.

$$r_c(s) = \frac{\mathcal{L}_c{}^2/k_c}{1+(\mathcal{R}_c/k_c)cos\theta} \tag{1.3}$$

The quantity $\varepsilon_c = \mathcal{R}_c/k_c$ is the eccentricity of the conic section and the angle θ is the angle between the vectors $\boldsymbol{e_r}$ and $\boldsymbol{\mathcal{R}_c}$.

Linearization of ellipses and production of Maxwell's equations in vacuum

When ellipses tend to become line segments, i.e. segments of geodesics of R^2, then the vectors $\boldsymbol{r}_c(s), \boldsymbol{u}_c(s)$ of a point of an ellipse are almost parallel. The inclination of the vectors $\boldsymbol{r}_c(s), \boldsymbol{u}_c(s)$ relative to the major axis of this ellipse is changing continuously during the linearization process. This inclination is mapped in R^3 with the gradient operator ($\boldsymbol{\nabla}$, **grad**) as follows: When $\boldsymbol{r}_c(s), \boldsymbol{u}_c(s)$ are located on the left side of each term of relations (1.1), (1.2), then they correspond to $\boldsymbol{\nabla}$. If the vectors $\boldsymbol{r}_c(s), \boldsymbol{u}_c(s)$ are located to the right side of those terms of relations (1.1), (1.2), then they are arguments of $\boldsymbol{\nabla}$, so they are defined as vector fields of R^3. The vectors $\boldsymbol{r}_c(s), \boldsymbol{u}_c(s)$ are defined in this way in order for the operator $\boldsymbol{\nabla}$ to act on a scalar or a vector quantity (by its cross product). So, we get

$$\boldsymbol{r_c} \leftrightarrow \boldsymbol{\nabla} \tag{2.1a}$$

$$\boldsymbol{u_c} \leftrightarrow \boldsymbol{\nabla} \tag{2.1b}$$

when $\boldsymbol{r_c}, \boldsymbol{u_c}$ are in the left side and

$$\boldsymbol{r_c} \leftrightarrow \mathbf{r} \tag{2.1c}$$

$$\boldsymbol{u_c} \leftrightarrow \boldsymbol{v} \tag{2.1d}$$

when $\boldsymbol{r_c}, \boldsymbol{u_c}$ are in the right side of terms of the relations (1.1), (1.2). The quantities $\mathbf{r}, \boldsymbol{v}$ are vector fields in R^3. Also, do the correspondences of the vectors $\boldsymbol{\mathcal{L}}_c, \boldsymbol{\mathcal{R}}_c$ and the constant k_c to the vector fields $\boldsymbol{L}, \boldsymbol{R}$ and to constant k of R^3 respectively, i.e.

$$\boldsymbol{\mathcal{L}}_c \leftrightarrow \boldsymbol{L} \tag{2.2a}$$

$$\boldsymbol{\mathcal{R}}_c \leftrightarrow \boldsymbol{R} \tag{2.2b}$$

$$k_c \leftrightarrow \mathrm{k} \tag{2.2c}$$

According to these correspondences, relations (1.1), (1.2) become relations that satisfy the holograms of R^3, having shadow-generators conics, namely

$$\boldsymbol{L} = \boldsymbol{\nabla} \times \boldsymbol{v} \tag{2.3a}$$

$$\boldsymbol{R} = \boldsymbol{\nabla} \times \boldsymbol{L} - \boldsymbol{\nabla}\left(\frac{\mathrm{k}}{\mathrm{r}}\right) \tag{2.3b}$$

where r is the magnitude of $\mathbf{r}$. The magnitude of $\boldsymbol{R}$ is determined by

$$R = |\boldsymbol{R}| = \mathrm{k} \tag{2.3c}$$

because of the mapping in R^3 of the relation

$$\varepsilon_c = \mathcal{R}_c / k_c \to 1^- \tag{2.3d}$$

We will examine the changes of the vector fields $\boldsymbol{L}$ and $\boldsymbol{R}$ in correspondence with the changes of $\boldsymbol{R}_c, \boldsymbol{L}_c$ with respect to s or with respect to the coordinate x of a point of an ellipse, because s and x are almost equal, during the linearization process of this ellipse. In order to find how the vector fields $\boldsymbol{L}$ and $\boldsymbol{R}$ will change, the correspondence between an open conic and a light wave will be used. Thus, from the formalistic similarity between the equation of a wave-front of a spherical light wave and the equation of a parabola, with respect to a coordinate system, which has the origin at its focus, we get the relations

$$ct = R \tag{2.4a}$$

or

$$(ct)^2 - R^2 = 0 \tag{2.4b}$$

The equation of a parabola is written as

$$x = \frac{1}{2a} y^2 \tag{2.4c}$$

or

$$x_0^2 - y^2 = 0 \tag{2.4d}$$

where $x_0^2 = 2ax$. From equations (2.4d), (2.4b), we do the correspondence of the coordinate x_0 of a point of a parabola with the distance ct of a point of the front of a spherical light-wave, from the center of the sphere with radius R appears directly. So, we get

$$x_0 \leftrightarrow ct \tag{2.4e}$$

or

$$x = \left(\frac{x_0}{2a}\right) x_0 \leftrightarrow \lambda ct \tag{2.4f}$$

or

$$s \cong x \leftrightarrow ct \qquad \text{for} \quad \lambda = \frac{x_0}{2a} \to 1 \tag{2.4g}$$

Equation (2.4g) allows us, to do correspondences between variations of geometrical quantities of conic sections in R^2 with variations of quantities of holograms in R^3. The vector fields $\boldsymbol{L}$ and $\boldsymbol{R}$, which are defined from equations (2.3a), (2.3b), will be the basis of a model, which interprets all the phenomena of Electromagnetism in vacuum.

a) Change of $\boldsymbol{L}$ with respect to ct

Differentiating the equation (2.3a) with respect to ct, we obtain

$$\frac{\partial \boldsymbol{L}}{\partial(\mathrm{ct})} = \boldsymbol{\nabla}\times \frac{\partial \boldsymbol{v}}{\partial(\mathrm{ct})} \tag{2.5a}$$

We define the isotropic vector fields in R^3, compatible to isotropy of R^3, as

$$\mathbf{B} = \frac{\partial \boldsymbol{L}}{\partial(\mathrm{ct})},\ \ \mathbf{A} = \frac{\partial \boldsymbol{v}}{\partial(\mathrm{ct})} \tag{2.5b}$$

The vector fields $\mathbf{B}$, $\mathbf{A}$ are called **magnetic field** and **vector potential**, respectively. Because of relations (2.5b), equation (2.5a) is written as

$$\mathbf{B} = \boldsymbol{\nabla}\times \mathbf{A} \tag{2.6}$$

From equation (2.6) and the identity $\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\times \mathbf{A}) = 0$ we get

$$\boldsymbol{\nabla}\cdot \mathbf{B} = 0 \tag{2.7}$$

Differentiating $\mathbf{B}$ with respect to ct, we obtain

$$\frac{\partial \mathbf{B}}{\partial(\mathrm{ct})} = \boldsymbol{\nabla}\times\left(\frac{\partial \mathbf{A}}{\partial(\mathrm{ct})} + \boldsymbol{\nabla}\Psi\right) \tag{2.8}$$

where Ψ is an arbitrary scalar function of R^3 and $\boldsymbol{\nabla}\times\boldsymbol{\nabla}\Psi = 0$. We define the isotropic vector field $\boldsymbol{E}$, compatible with the isotropy of R^3, as

$$\boldsymbol{E} = -\left(\frac{\partial \mathbf{A}}{\partial(\mathrm{ct})} + \boldsymbol{\nabla}\Psi\right) \tag{2.9}$$

This vector field is called the **electric field**. Due to (2.9), equation (2.8) is written as

$$\frac{\partial \mathbf{B}}{\partial(\mathrm{ct})} = -\boldsymbol{\nabla}\times\boldsymbol{E} \tag{2.10}$$

The quantities which are called **magnetic field, vector potential** and **electric field** of the holograms satisfy relations (2.5b), (2.6), (2.7), (2.9), (2.10), which are formalistically similar to respective relations of electromagnetic theory developed in R^3, through a systematic classification of experimental data by Maxwell.

b) Change of $\boldsymbol{R}$ with respect to ct

When $\varepsilon_c = 1$, then from the (2.3a) we get $\boldsymbol{L}_c = 0$, because the $\boldsymbol{r}_c(s), \boldsymbol{u}_c(s)$ are precisely parallel vectors. Mapping this relationship in R^3, we get the relation

$$\boldsymbol{L} = 0 \tag{2.11}$$

In view of relation (2.11), the equations (2.5) and (2.3b) are written as

$$\mathbf{B} = 0 \tag{2.12a}$$

$$\boldsymbol{R} = -\boldsymbol{\nabla}\left(\frac{\mathrm{k}}{\mathrm{r}}\right) \tag{2.12b}$$

From equations (2.6) and (2.12a) we obtain

$$\mathbf{A} = 0 \ \text{ or } \ \mathbf{A} = \boldsymbol{\nabla}\tilde{\Phi} \tag{2.13}$$

where $\tilde{\Phi}$ is a scalar function of R^3. From (2.13) and the definition of the electric field $\boldsymbol{E}$, we get

$$\boldsymbol{E} = -\,\boldsymbol{\nabla}\Phi \tag{2.14}$$

where

$$\Phi = \Psi \ \text{ or } \ \Phi = \frac{\partial\tilde{\Phi}}{\partial(\mathrm{ct})} + \Psi \tag{2.14'}$$

For $\varepsilon_c \to 1^-$, the shadow-generators of the vector fields $\boldsymbol{R}, \boldsymbol{E}$ of R^2, are two parallel vectors. Due to one-to-one correspondences, between the shadow-generators of R^2 and the holograms in R^3, as well as the conservation of the concept of parallelism between the flat spaces R^2 and R^3, the vector fields $\boldsymbol{R}, \boldsymbol{E}$, are parallel in R^3, as well as the shadow-generators are in R^2. Choosing an appropriate value of Φ, the vector fields $\boldsymbol{R}, \boldsymbol{E}$ become equal, i.e.

$$\boldsymbol{R} = \pm\boldsymbol{E} \tag{2.15}$$

The $\pm$ sign means that $\boldsymbol{R}, \boldsymbol{E}$ are parallel or antiparallel vector fields. Substituting the (2.12b) and (2.14), to the (2.15), is taken

$$\boldsymbol{\nabla}\left(\frac{\mathrm{k}}{\mathrm{r}}\right) = \pm\,\boldsymbol{\nabla}\Phi \tag{2.16}$$

or

$$\Phi = \pm\frac{\mathrm{k}}{\mathrm{r}} \ \text{ and } \ \boldsymbol{E} = \pm\frac{\mathrm{k}}{\mathrm{r}^3}\mathbf{r} \tag{2.17}$$

Because of equation (2.17) and the fact that $\boldsymbol{E}$ is an isotropic vector, the flow of $\boldsymbol{E}$ through a sphere of radius r, is given by the relation

$$\oint_{\partial S} \boldsymbol{E} \cdot \boldsymbol{dS} = \pm 4\pi\mathrm{k} \tag{2.18}$$

where $\oint_{\partial S} d\Omega = 4\pi$ and $d\Omega$ is the solid angle element. We define the scalar quantity q proportional to the flow of $\boldsymbol{E}$ through this sphere as

$$q = \varepsilon_0(\pm 4\pi\mathrm{k}) \tag{2.19}$$

where ε_0 is a proportionality constant. We call **charge,** the holographic quantity q which is not a fundamental quantity in R^3. It takes positive or negative values, due to fact that $\boldsymbol{R}, \boldsymbol{E}$ are parallel or antiparallel vector fields, as their shadow-generators. Substituting k from the equation (2.19), to the (2.17), we take

$$\Phi = \frac{q}{4\pi\varepsilon_0 \mathrm{r}} \tag{2.20}$$

$$\boldsymbol{E} = \frac{q}{4\pi\varepsilon_0 \mathrm{r}^3}\mathbf{r} \tag{2.21}$$

where the $\pm$ sign is incorporated into q. Assuming that the charge q is uniformly distributed in the volume V of the sphere with density ρ, i.e. $q = \rho V$, then from the divergence theorem (Gauss's theorem), we get

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0} \tag{2.22}$$

The quantity we have called charge satisfies relation (2.22) that is formalistically similar to Poisson's equation. Thus, the quantity q matches with the physical quantity which is also called charge.

The existence of charge q, was initially verified through observations of Thales of Miletus and was experimentally confirmed centuries later by J.J. Thomson. The charge was regarded as a fundamental real physical quantity of R^3, but of unknown origin. Charge can be regarded as a quantity that defines the vector fields $\boldsymbol{E}$ and $\boldsymbol{B}$. Through those fields and the relations that they satisfy, all electromagnetic phenomena are interpreted. But the appearance of a positive or negative sign of charge has not been explained in a satisfactory and convincing way. However, in the framework of mappings of shadow-generators to holograms, the existence of two signs of charge is interpreted. This interpretation is based on the fact that $\boldsymbol{R}$, $\boldsymbol{E}$ are parallel or antiparallel vector fields because their shadow-generators are parallel or antiparallel vectors during the linearization process of an ellipse.

We will finally consider the variation of the total vector field $\boldsymbol{R}$ in ct. Differentiating equation (2.3b) with respect to ct we get

$$\frac{\partial \boldsymbol{R}}{\partial(\mathrm{ct})} = \boldsymbol{\nabla}\times \frac{\partial \boldsymbol{L}}{\partial(\mathrm{ct})} - \frac{\partial}{\partial(\mathrm{ct})}\boldsymbol{\nabla}\left(\frac{\mathrm{k}}{\mathrm{r}}\right) \tag{2.23}$$

From the definitions (2.5), (2.14), equation (2.23), is expressed as

$$\frac{\partial R}{\partial(\mathrm{ct})}\boldsymbol{e}_{\mathbf{R}} = \boldsymbol{\nabla}\times\mathbf{B} + \frac{\partial \boldsymbol{E}}{\partial(\mathrm{ct})} \tag{2.23'}$$

Substituting (2.3) and (2.19) in the above equation, this is written as

$$\frac{\partial}{\partial(\mathrm{ct})}\left[\left(\frac{q}{4\pi\varepsilon_0}\right)\boldsymbol{e}_{\mathbf{R}}\right] - \frac{\partial \boldsymbol{E}}{\partial(\mathrm{ct})} = \boldsymbol{\nabla}\times\mathbf{B} \tag{2.23''}$$

Selecting the negative sign for charge ($q < 0$) in the previous relationship, we obtain

$$\boldsymbol{\nabla}\times\mathbf{B} = \left[\frac{1}{4\pi\varepsilon_0}\left(-\frac{\partial|q|}{\partial(\mathrm{ct})}\boldsymbol{e}_{\mathbf{R}}\right)\right] + \frac{\partial \boldsymbol{E}}{\partial(\mathrm{ct})} \tag{2.24}$$

Defining the vector field $\boldsymbol{J}$ of R^3 as

$$\boldsymbol{J} = -\frac{1}{4\pi\varepsilon_0}\left(\frac{\partial\rho}{\partial t}\hat{\boldsymbol{e}}_{\boldsymbol{R}}\right) \tag{2.25}$$

The vector field $\boldsymbol{J} = \frac{\boldsymbol{I}}{V}$ with $q = \rho V$ is called **current density** $\boldsymbol{I}$, where $\boldsymbol{I} = \frac{1}{4\pi\varepsilon_0}\left(\frac{\partial|q|}{\partial t}\hat{\boldsymbol{e}}_{\boldsymbol{R}}\right)$. Due to relation (2.25), equation (2.24) is written as

$$\nabla\times\mathbf{B} = \frac{\partial\boldsymbol{E}}{c\,\partial t} + \frac{\boldsymbol{J}}{c} \tag{2.26}$$

The term $\frac{\boldsymbol{J}}{c}$ of the right hand side of (2.26) is formalistically similar to the "**displacement current**", that Maxwell introduced arbitrarily, in his equations, in order for them to be compatible with the experimental data.

Equations (2.7), (2.10), (2.22), (2.26) are formalistically similar to the **Maxwell equations in vacuum**. All those equations are gathered together, so we assume

First couple (variation of $\boldsymbol{L}$): $\quad \nabla\cdot\mathbf{B} = 0 \quad , \quad \frac{\partial\mathbf{B}}{\partial(ct)} = -\nabla\times\boldsymbol{E} \qquad (2.27a)$

Second couple: (variation of $\boldsymbol{R}$): $\nabla\cdot\boldsymbol{E} = \frac{\rho}{\varepsilon_0} \quad , \quad \nabla\times\mathbf{B} = \frac{\partial\boldsymbol{E}}{c\,\partial t} + \frac{\boldsymbol{J}}{c} \qquad (2.27b)$

with

$$\mathbf{B} = \nabla\times\mathbf{A}\ , \quad \boldsymbol{E} = -\left(\frac{\partial\mathbf{A}}{\partial(ct)} + \nabla\Psi\right) \tag{2.27c}$$

The equations (2.27) are the fundamental equations on which the **Electromagnetic theory in vacuum** is based. The second pair of (2.27), that is produced from the variation of the vector field $\boldsymbol{R}$, contains the "**sources**" ρ and $\boldsymbol{J}$. This observation lead us to generalize relations (2.3), by differential forms in R^3, using of a different type of "source", as for example a "**mass-source**".

Production of the wave equations

As already mentioned, through the linearization process of ellipses, those ellipses have, the same eccentricity with a parabola. So, a linear parabola is regarded as the limit curve of linear ellipses. A parabola corresponds to a wave in R^3, so equation (2.4b) of a parabola, is formalistically similar to the equation of a wave. Also, the equations of Maxwell are obtained, through the mapping in R^3, from the linearization process of ellipses in R^2. When the equations of Maxwell combine, then the result is a wave equation that is formalistically similar to the equation of a parabola. This formal similarity will lead us to the discovery of new physical entities in R^3.

We will start from the simple case where $\rho = 0$ and $\boldsymbol{J} = 0$ finding the conditions under which the combination of Maxwell's equations gives a wave equation that is formalistically similar to the equation of a parabola.
From equation $\nabla\times\mathbf{B} = \frac{\partial \boldsymbol{E}}{c\,\partial \mathrm{t}}$ and the definitions $\mathbf{B} = \nabla\times \mathbf{A}$, $\boldsymbol{E} = -\left(\frac{\partial \mathbf{A}}{\partial\,(\mathrm{ct})} + \nabla\Psi\right)$ we get

$$\nabla\times(\nabla\times\mathbf{A}) = -\frac{\partial^2 \mathbf{A}}{c^2\,\partial^2 \mathrm{t}} - \nabla\frac{\partial \Psi}{\mathrm{c}\,\partial(\mathrm{t})} \tag{3.1a}$$

Because of the vector identity $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$ we assume

$$\nabla^2\mathbf{A} - \frac{\partial^2 \mathbf{A}}{c^2\,\partial^2 \mathrm{t}} = \nabla\left(\nabla\cdot\mathbf{A} + \frac{\partial \Psi}{\mathrm{c}\,\partial(\mathrm{t})}\right) \tag{3.1b}$$

To be formalistically similar, equation (3.1b), with equation (2.4d) of a parabola, then the right member must be zero, i.e. the quantity in parentheses must be a constant. Choosing the zero value for this constant, we obtain the equation

$$\nabla\cdot\mathbf{A} + \frac{\partial \Psi}{\mathrm{c}\,\partial(\mathrm{t})} = 0 \tag{3.2}$$

Equation (3.2) is called **Lorentz gauge** or **Lorentz gauge condition**. Thus, (3.1b) due to (3.2), is the homogeneous wave equation with group velocity $\mathbf{c}$, i.e. the equation of a light wave

$$\nabla^2\mathbf{A} - \frac{\partial^2 \mathbf{A}}{c^2\,\partial^2 \mathrm{t}} = 0 \tag{3.3}$$

A corresponding equation is taken for the scalar potential Φ, resulting from $\nabla\cdot\boldsymbol{E} = 0$ and the Lorenz gauge , i.e.

$$\nabla^2\Phi - \frac{\partial^2 \Phi}{c^2\,\partial^2 \mathrm{t}} = 0 \tag{3.4}$$

Following the same procedure but with $\rho \neq 0$ and $\boldsymbol{J} \neq 0$ we get the non-homogeneous wave equations

$$\nabla^2\mathbf{A} - \frac{\partial^2 \mathbf{A}}{c^2\,\partial^2 \mathrm{t}} = -\,{}^{\boldsymbol{J}}\!/_{\varepsilon_0} \tag{3.5a}$$

$$\nabla^2\Phi - \frac{\partial^2 \Phi}{c^2\,\partial^2 \mathrm{t}} = -\,{}^{\rho}\!/_{\varepsilon_0} \tag{3.5b}$$

Equations (3.5) are formalistically similar to the equation of an equilateral hyperbola or equivalently to the equation of a parabola with respect to a coordinate system which has its origin at a point of the symmetry axis of the parabola between its focus and its directrix. The process of transforming a closed conic section to an open one, in the eccentricities region $(1^{-},1]$, is illustrated in R^3 with physical entities, that combine the corpuscular and wave features, which are the corresponding features of closed and open conic sections respectively.

The uncertainty about the type of a conic section in this range of eccentricities or else lack of knowledge about the type of conic sections is illustrated in R^3 as a lack of knowledge of the observer about the exact position of the particle due to the wave nature of those physical entities. The existence of physical entities in R^3different from pure bodies and pure waves, has verified experimentally. Those particles are called **waveparticles** or **wave quanta**, because of duality that they exhibit, having simultaneously the general features of bodies and waves. Through this way of thinking, Quantum Mechanics is developed, so these physical entities can be investigated in detail.

Maxwell equations in a dielectric medium

Relations

$$\boldsymbol{L} = \boldsymbol{\nabla}\times\boldsymbol{v} \tag{2.3a}$$

$$\boldsymbol{R} = \boldsymbol{\nabla}\times\boldsymbol{L} - \boldsymbol{\nabla}\left(\frac{\mathrm{k}}{\mathrm{r}}\right) \tag{2.3b}$$

are the relations through which vector fields $\boldsymbol{L}$ and $\boldsymbol{R}$ are defined on R^3, leading to Maxwell's equations in vacuum as well wave equations as we saw above. These relations will be generalized, writing them as relations between differential forms. The reason we make this generalization is to find their expressions on R^3, and then to have them transferred to a curved differential manifold. Let $u^1, u^2, \dots, u^n$ be curvilinear coordinates in a neighborhood of a point of a manifold V. From these we take the position vector $\boldsymbol{r}$, its derivatives which spanning a basis in this neighborhood, and the inner product of two random vectors of this basis, i.e.

$$\boldsymbol{r} = \boldsymbol{r}(u^1, u^2, \dots, u^n)$$

$$\boldsymbol{e}_i = \frac{\partial \boldsymbol{r}}{\partial u^\iota}$$

$$g_{ij} = \left(\boldsymbol{e}_i, \boldsymbol{e}_j\right) = \boldsymbol{e}_i \cdot \boldsymbol{e}_j$$

$$g^{\iota j} = \left(g_{ij}\right)^{-1}$$

g_{ij}, are the coefficients of the metric that allow us to work with distances and angles in the coordinate system $u^1, u^2, \dots, u^n$. The vectors $\boldsymbol{e}_i$, are basis vectors of a point with position vector $\boldsymbol{r}$. Each vector $\boldsymbol{v}$, which begins at this point, is analyzed to the base $(\boldsymbol{e}_i)$, as follows

$$\boldsymbol{v} = v^i \boldsymbol{e}_i$$

In order to interpret v^i, we sought a function b^i, that has as argument the vector $\boldsymbol{v}$ and the result of the action of b^i on $\boldsymbol{v}$ is its component v^i, i.e.

$$b^i(\boldsymbol{v}) = b^i\left(v^j \boldsymbol{e}_j\right) = v^j b^i\left(\boldsymbol{e}_j\right) = v^i$$

In order for the previous relationship to be correct, relation $b^i\left(\boldsymbol{e}_j\right) = \delta^i_j$ should be applied. This means that b^i is the base element of a base which is **dual to the base**, which has as a base element the vector $\boldsymbol{e}_j$. This base has as elements the 1-forms du^i. Since $du^i: V \to R$ then all du^i belong to the dual space V^* where V^* is the vector space of linear functionals on V. We define the function $\Phi: V \to V^*$ and its inverse $\Phi^{-1}: V^* \to V$ as follows

$$\Phi(\boldsymbol{v}) = g_{ij} v^i du^j$$

$$\Phi^{-1}(\omega) = g^{ij} \omega_i \boldsymbol{e}_j$$

Function $\boldsymbol{\Phi}$ has a vector as an argument and value a 1-form and function $\boldsymbol{\Phi}^{-1}$ has a 1-form as an argument and value a vector. The symbol $*$ (**Hodge star**) is an operator that is defined on a n-dimension space V as follows

$$*: \Lambda^r(V) \to \Lambda^{n-r}(V)$$

where $\Lambda^r(V)$ is the set of all r-forms in V. From the general definition of the $grad$ and the $curl$ [2], we get

$$curl = \Phi^{-1} \circ * \circ\, d \circ \Phi$$

$$grad = \Phi^{-1} \circ d$$

The operator d has the following properties:
(1) It is linear
(2) For $f \in \Lambda^0(V)$ then $df = \frac{\partial f}{\partial v^i} dv^i$ where $\Lambda^0(V)$ is the set of all functions of V
(3) If $\omega \in \Lambda^j(V)$ and $\eta \in \Lambda^k(V)$ then $d(\omega \wedge \eta) = d\omega \wedge \eta + (-1)^j \omega \wedge d\eta$ (Leibniz rule)
(4) $dd = 0$ (Poincaré's Lemma: if ω is a differential form then $dd\omega = 0$)

The symbol $\circ$ denotes the composition of functions $d, \Phi, *$. The quantities $d, \Phi, *$ and their compositions, that are expressed through $curl$ and $grad$, are independent of the choice of coordinate systems. We write relations (2.3a), (2.3b) as follows:

$$\boldsymbol{L} = curl(\boldsymbol{v})$$

$$\boldsymbol{R} = curl(\boldsymbol{L}) + grad(\Phi)$$

According to the definitions of $curl$, $grad$, we get

$$\boldsymbol{L} = \Phi^{-1} * d\Phi(\boldsymbol{v})$$

$$\boldsymbol{R} = \Phi^{-1} * d\Phi(\boldsymbol{L}) + \Phi^{-1} d(\Phi)$$

where $\Phi = \frac{\mathrm{k}}{\mathrm{r}} \in \Lambda^0(V)$. We act on the two members of the above relations from the left, with the function $\boldsymbol{\Phi}$, so we get

$$\Phi(\boldsymbol{L}) =* d\Phi(\boldsymbol{v})$$

$$\Phi(\boldsymbol{R}) =* d\Phi(\boldsymbol{L}) + d(\Phi)$$

We act from the left, with the operator $*$ on the above relations, so we get

$$* \Phi(\boldsymbol{L}) =** d\Phi(\boldsymbol{v})$$

$$* \Phi(\boldsymbol{R}) =** d\Phi(\boldsymbol{L}) +* d(\Phi)$$

From the general relation

$$** \omega = (-1)^{r(n-r)}\omega, \text{ with } \omega \in \Lambda^r(V^*)$$

where $\omega = d\Phi(\boldsymbol{v})$, and the $d\Phi(\boldsymbol{v})$ belongs to the set $\Lambda^2(V)$, we get

$$* \Phi(\boldsymbol{L}) = d\Phi(\boldsymbol{v})$$

$$* \Phi(\boldsymbol{R}) = d\Phi(\boldsymbol{L}) +* d(\Phi)$$

We define the forms

$$\mathrm{L} =* \Phi(\boldsymbol{L}),\ \mathrm{R} =* \Phi(\boldsymbol{R}),\ \breve{\mathrm{D}} =* d(\Phi)$$

$$\mathrm{u} = \Phi(\boldsymbol{v}),\ \mathrm{V} = \Phi(\boldsymbol{L})$$

From the above definitions, relations (2.3a), (2.3b) are written through differential forms, as

$$\mathrm{L} = \mathrm{du} \tag{4.1}$$

$$\mathrm{R} = \mathrm{dV} + \breve{\mathrm{D}} \tag{4.2}$$

The $\mathrm{L}, \mathrm{R}, \breve{\mathrm{D}}$ are 2-forms, i.e. belonging to the set $\Lambda^2(V)$, and u, V are 1-forms, i.e. belonging to the set $\Lambda^1(V)$.

In R^3, we can use the natural coordinate system (x^i) or any other curvilinear coordinate system (v^i), where the v^i are not necessarily rectangular. The time evolution of the forms L and R will be found, and from them the general equations of Maxwell will be taken, i.e. the equations of Maxwell in any dielectric medium. Differentiating (4.1) with respect to ct, as in the paragraph 2, we take

$$\frac{\partial \mathrm{L}}{c\partial t} = d\left(\frac{\partial u}{c\partial t}\right) \tag{4.3}$$

We define the 1-form A and the 2-form B as

$$A = \frac{\partial u}{c\partial t} \tag{4.4}$$

$$B = \frac{\partial \mathrm{L}}{c\partial t} \tag{4.5}$$

Because of (4.4) and (4.5), equation (4.3), is written as

$$B = dA \tag{4.6}$$

From the property, $dd = 0$, of the operator d, we get from (4.6) the relationship

$$dB = 0 \tag{4.7}$$

The time evolution of the 2-form B, is taken, differentiating the relation (4.6) with respect to ct, so we take

$$\frac{\partial B}{c\partial t} = d\left(\frac{\partial A}{c\partial t} + d\Psi\right) \tag{4.8}$$

where $\Psi \in \Lambda^0(\mathrm{R}^3)$. Defining the 1-form E as

$$E = -\left(\frac{\partial A}{c\partial t} + d\Psi\right) \tag{4.9}$$

Because of (4.9), the equation (4.8) is written as

$$dE = -\frac{\partial B}{c\partial t} \tag{4.10}$$

Applying the general Stokes theorem for the 2-form $\tilde{\mathrm{D}}$, in the (4.2), in a closed manifold M of R^3, we take

$$\oint_{\partial M} \tilde{\mathrm{D}} = \oint_{M} \mathrm{d}\tilde{\mathrm{D}} \tag{4.11}$$

The surface integral $\oint_{\partial M} \tilde{\mathrm{D}}$, is a scalar quantity, whose value is the same, if it is calculated through vector fields or through forms. So, from the equations (2.18), (2.19) we get the relation

$$\oint_{\partial M} \tilde{\mathrm{D}} = \frac{q}{\varepsilon_0} \tag{4.12}$$

where q has positive or negative sign. If the density of q in the volume V of M is ρ, then

$$q = \oint_M \rho \Omega_0 \tag{4.13}$$

where Ω_0 is the **top-form** of M. If u^i are generally n curvilinear coordinates at a point of a manifold M and g is the determinant of the metric of a neighborhood of this point, then the definition of the top-form Ω_0 is

$$\Omega_0 = \sqrt{g} du^1 \wedge du^2 \wedge \ldots \wedge du^n \tag{4.13'}$$

Substituting relation (4.13) to (4.12), we obtain

$$\oint_{\partial M} \tilde{\mathrm{D}} = \frac{q}{\varepsilon_0} = \oint_M \frac{\rho}{\varepsilon_0} \Omega_0 \tag{4.14}$$

Since the manifold M is a closed surface embedded in R^3 then the$\tilde{\mathrm{D}}$, as a 2-form of R^3, is written

$$\tilde{\mathrm{D}} = \tilde{\mathrm{D}}_x dydz + \tilde{\mathrm{D}}_y dzdx + \tilde{\mathrm{D}}_z dxdy \tag{4.14'}$$

Substituting this expression of $\tilde{\mathrm{D}}$ in the right member of the general theorem of Stokes, we get

$$\oint_M \mathrm{d}\tilde{\mathrm{D}} = \oint_M \left(\frac{\partial \tilde{\mathrm{D}}_x}{\partial x} + \frac{\partial \tilde{\mathrm{D}}_y}{\partial y} + \frac{\partial \tilde{\mathrm{D}}_z}{\partial z} \right) dxdydz = \oint_M \mathrm{d}\tilde{\mathrm{D}} \Omega_0 \tag{4.15}$$

Equating the last members of (4.14) and (4.15), due to the general theorem of Stokes, we get for the divergence of $\tilde{\mathrm{D}}$, the following equation

$$\mathrm{d}\tilde{\mathrm{D}} = \frac{\rho}{\varepsilon_0} \tag{4.16}$$

Equation (4.16) is the equation of Poisson, which is written through forms, and expresses the relation between the divergence of the charge-source $\tilde{\mathrm{D}}$, and the charge density. The time variation of R is obtained, differentiating both members of (4.2), with respect to ct, i.e.

$$\frac{\partial \mathrm{R}}{c \partial t} = \mathrm{d} \left(\frac{\partial \mathrm{V}}{c \partial t} \right) + \frac{\partial \tilde{\mathrm{D}}}{c \partial t} \tag{4.17}$$

We define the 1-form H and the 2-form J as

$$H = \frac{\partial \mathrm{V}}{c \partial t} \tag{4.18}$$

$$J = \frac{\partial \mathrm{R}}{\partial t} \tag{4.19}$$

Substituting relations (4.18), (4.19), to (4.17), and taking negative or positive charge, i.e. ($\tilde{\mathrm{D}} < 0$) or ($\tilde{\mathrm{D}} > 0$), the equation (4.17) is written as

$$\mathrm{d}H = \frac{J}{c} \pm \frac{\partial \tilde{\mathrm{D}}}{c\partial t} \tag{4.20}$$

Equations (4.6), (4.10), (4.16) and (4.20), are Maxwell's equations, which are expressed through differential forms of R^3. Maxwell's equations are produced, in a very simple way through this process, and they are applicable to any medium because of the appearance of the quantities H and $\tilde{\mathrm{D}}$. The quantities H and $\tilde{\mathrm{D}}$, are not identical to B and E respectively, as in the case that the quantities L, R are vector fields of R^3 in vacuum. In order to be associated the H, $\tilde{\mathrm{D}}$ with the B, E, we assume the relations

$$\tilde{\mathrm{D}} = \varepsilon(* E) \tag{4.20΄a}$$

$$B = \mu(* H) \tag{4.20΄b}$$

The quantities ε, μ are interpreted as **dielectric constant** and **magnetic permeability** respectively, of a medium other than the vacuum. Maxwell's equations suggests that $\tilde{\mathrm{D}}$ and B should be 2 forms and E and H should be 1-forms. Since, the $E, H \in \Lambda^1(R^3)$, then the $* E, * H \in \Lambda^2(R^3)$, i.e. they are 2-forms. We also note that $\tilde{\mathrm{D}}$ and E are not trivial variants of one another. This observation expressed by Maxwell and Faraday. Charge results from the flux of E. Since, the $\tilde{\mathrm{D}}$ is correlated to the E, then the $\tilde{\mathrm{D}}$ is considered as **a source of positive or negative charge**. This remark opens the way for general equations to be found, assuming that the $\tilde{\mathrm{D}}$, is a mass-source with $\tilde{\mathrm{D}} > 0$. Through this way of thinking and the development of General Relativity, we shall investigate in detail physical phenomena in a large scale.

The conclusion coming from the above paper is that through the equations on which electromagnetic theory is founded, we can unify physical phenomena at any scale. Since the electromagnetic theory through Quantum Mechanics, leads us to explore Nature in tiny scale and through General Relativity to explore Nature in large scale then all physical phenomena can be unified at any scale.

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